Government of Western Australia
School Curriculum and Standards Authority

MATHEMATICS: NUMBER AND ALGEBRA, MEASUREMENT AND GEOMETRY

Teaching, Learning and Assessment Exemplar

Year 9

Pythagoras' TV-Rem

DRAFT

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

Contents

Background

This Teaching, Learning and Assessment Exemplar (the exemplar) has been developed by the School Curriculum and Standards Authority (the Authority) as part of the *School Education Act Employees (Teachers and Administrators) General Agreement 2017* (Clause 61.1–61.3).

The *Western Australian Curriculum and Assessment Outline* (the *Outline* – https://k10outline.scsa.wa.edu.au/) sets out the mandated curriculum, guiding principles for teaching, learning and assessment, and support for teachers in their assessment and reporting of student achievement. The *Outline* recognises that all students in Australian schools, or international schools implementing the Western Australian curriculum, are entitled to be given access to the eight learning areas described in the *Alice Springs* (*Mparntwe*) *Education Declaration*, December 2019.

The Mathematics Pythagoras and Coordinate Geometry exemplar for Year 9 articulates the curriculum in the *Outline* and approaches to teaching, learning and assessment reflective of the Principles of Teaching, Learning and Assessment. This exemplar presents a sequence of teaching and learning, including suggested assessment points, for 12 lessons, with a time allocation of three hours per week.

Teaching

The year-level syllabuses for each learning area deliver a sequential and age-appropriate progression of learning and have the following key elements:

- a year-level description that provides an overview of the context for teaching and learning in the year
- a series of content descriptions, populated through strands and sub-strands, that sets out the knowledge, understanding and skills that teachers are expected to teach and students are expected to learn
- an achievement standard describes an expected level that the majority of students are achieving by the end of a given year of schooling. An achievement standard describes the quality of learning (e.g. the depth of conceptual understanding and the sophistication of skills) that indicate the student is well-placed to commence the learning required in the next year.

Assessing

Assessment, both formative and summative, is an integral part of teaching and learning. Assessment should arise naturally out of the learning experiences provided to students. In addition, assessment should provide regular opportunities for teachers to reflect on student achievement and progress. As part of the support it provides for teachers, this exemplar includes suggested assessment points. It is the teacher's role to consider the contexts of their classroom and students, the range of assessments required, and the sampling of content descriptions selected to allow their students the opportunity to demonstrate achievement in relation to the year-level achievement standard.

Reflecting

Reflective practice involves a cyclic process during which teachers continually review the effects of their teaching and make appropriate adjustments to their planning. The cycle involves planning, teaching, observing, reflecting and replanning.

Teachers may choose to expand or contract the amount of time spent on developing the required understandings and skills according to their reflective processes and professional judgements about their students' evolving learning needs.

Catering for diversity

This exemplar provides a suggested approach for the delivery of the curriculum and reflects the rationale, aims and content structure of the learning area. When planning the learning experiences, consideration has been given to ensuring that they are inclusive and can be used in, or adapted for, individual circumstances. It is the classroom teacher who is best placed to consider and respond to (accommodate) the diversity of their students. Reflecting on the learning experiences offered in this exemplar will enable teachers to make appropriate adjustments (where applicable) to better cater for students' gender, personal interests, achievement levels, socio-economic, cultural and language backgrounds, experiences and local area contexts.

At any point, teachers can adjust the:

- **timing of the lessons**, e.g. allowing more time where required, or changing when content is taught to fit school, local or cultural events, such as NAIDOC Week, school or interschool carnivals or major sporting events, such as the Olympic Games
- **scheduling of assessments** to allow for further consolidation of teaching and learning, or to fit with students' personal or cultural events, such as Ramadan
- **mode of delivery**, e.g. allowing students to present an oral report with pictures and models rather than a written one, or contributing to a digital presentation instead of a written submission or journal
- **setting of the lessons**, e.g. the school gymnasium or courts, visiting a local construction site or using mapping software to investigate parallel lines in town planning
- **opportunities to engage with the content descriptions,** e.g. emulating parallel lines on ClassPad emulators to introduce the technology and demonstrate relationships in action, linking parallel lines and angles to hand-writing, looking at famous railways international and through history
- **ways students work**, e.g. students supporting each other in mixed ability groups or teachers forming ability groups for targeted support
- **delivery of the content descriptions** to make it more engaging, challenging or appropriate, e.g. creating hands on concrete materials to demonstrate concepts, researching a person or event that is related to the content or utilising resources such as YouTube
- **teaching strategies used**, this exemplar utilises strategies such as multiple representations, collaborative learning opportunities and a range of opportunities for consolidation. These should be considered an example of possible ways of teaching which should be utilised depending on the capabilities and needs of the students' learning needs
- **content descriptions, skills or modes of learning for individuals** with formal or informal learning adjustments.

The general capabilities and cross-curriculum priorities

The *Outline* incorporates seven general capabilities and three cross-curriculum priorities that can be utilised to connect learning across the eight learning areas.

The general capabilities and cross-curriculum priorities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities and priorities into their teaching and learning programs.

The full description and exemplification of the general capabilities can be found on the Authority website: https://k10outline.scsa.wa.edu.au/home/teaching/general-capabilities-over/general-capabilities-overview/general-capabilities-in-the-australian-curriculum.

The full description and exemplification of the cross-curriculum priorities can be found on the Authority website: https://k10outline.scsa.wa.edu.au/home/teaching/cross-curriculum-priorities2/cross-curriculum-priorities.

Teaching Mathematics

The Western Australian Curriculum: Mathematics acknowledges that the study of Mathematics provides students with fundamental mathematics skills to support them in their personal, work and civic lives. The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem-solving skills. These capabilities enable students to respond to familiar and unfamiliar situations by employing a range of flexible mathematical strategies to make informed decisions and solve problems efficiently.

The three content strands of Number and Algebra, Measurement and Geometry, and Statistics and Probability together with the four proficiency strands of Problem-Solving, Reasoning, Understanding and Fluency define the mandated Western Australian Curriculum: Mathematics. The proficiencies describe how the content is to be explored and taught and should be developed simultaneously with the content. The four proficiency strands describe the actions of thinking and doing Mathematics. Together with the mathematical content, these actions become increasingly sophisticated as students progress from Pre-Primary to Year 10.

The structure of the mathematics curriculum allows for students to develop transferrable skills which foster a deeper understanding of interrelated and interdependent concepts beyond the classroom. The numeracy capabilities developed by students assist them to make meaning in all Learning Areas.

This teaching, learning and assessment exemplar provides a sequence of lessons that reflect an integration of the three content strands of Number and Algebra, Measurement and Geometry, and Statistics and Probability, in accordance to the 'Ways of Teaching' in Mathematics and the *Outline*. To ensure that all aspects of the mandated Year 9 strands and sub-strands are taught, refer to the Mathematics page of the Western Australian curriculum at: https://k10outline.scsa.wa.edu.au/home/teaching/curriculum-browser/mathematics-v8.

Diagram 1 – How to read the teaching and learning exemplar

Lesson 1: Review of coordinates

Western Australian curriculum content 1

- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point [Year 7 content]
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment 2	Suggested learning experiences 3 (Lesson 1)
Learning intentions 2a • Revise Year 7 content. • Plot coordinates on a Cartesian plane. • Write Cartesian coordinates for a point on a Cartesian plane. • Identify the four quadrants. **Focus questions** 2b • Where might we use coordinates in everyday life? • How can you remember which axis each coordinate relates to?	**Sequence** 1. Warm-up related to Cartesian coordinates using an activity such as Cartesian coordinates figures. In this activity, students place figures of different sizes and shapes on a Cartesian plane from −10 to 10. This can be modified to support students who are not strong with negative numbers by including either all coordinates in the first quadrant or using only the first quadrant and either the second or fourth quadrant. (Appendix A.2) 2. Write a pair of coordinates on the board. Students use a whiteboard or tablet to show 1, 2, 3 or 4 to represent which quadrant the coordinates are located. Use coordinates, such as $(0,0)$, $(0,4)$ and $(5,0)$, to invite critical thinking about which quadrant coordinates on the axis are (the definition states they are not in any quadrant). 3. Display a Cartesian plane on the board or projector, getting students to come up in groups of four or five to place a sticker on a given coordinate. Students write the coordinates of their sticker on the board, checking and recording their understanding as they go. Use this as a formative assessment opportunity to determine where the class is at with their understanding of plotting Cartesian coordinates.

1. The Western Australian curriculum is the mandated curriculum content to be taught from the *Outline*.
2. Teaching, learning intentions and assessment may provide additional information and/or examples to assist with the interpretation of curriculum content.
   a. The learning intentions are the high-level ideas involved in teaching students to think mathematically. They are drawn from the knowledge and understanding in the syllabus for each year.
   b. Focus questions scaffold the teaching and learning and are integral to the learning experiences.
   c. Suggested formative assessment opportunities provide prompts to monitor student progress and to facilitate teacher planning. (Not pictured)
3. The sequence and learning experiences describe the interaction and activities that take place to facilitate learning. The sequence suggests the order of learning experiences to support the learning intentions. Sample activities are referenced to give examples of how to develop understanding of the mandated curriculum content.

Ways of teaching

The teaching and learning opportunities offered in this exemplar are not exhaustive. Thus, teachers are encouraged to make professional decisions about which learning experiences suit the needs of their students and how these may be adopted, according to resources and context. Furthermore, the sequence and time spent on the lessons may also require adjustment to ensure authenticity of student learning and engagement. These sample lessons may prove a useful starting point for amplifying creativity in the classroom, while reflecting the embedded expectations of the Western Australian Curriculum: Mathematics.

Ways of teaching	Possible learning experiences
Use appropriate materials, models, images or other representations purposefully to support students to move towards abstract ideas and create new knowledge and strategies	• practical, hands-on activities • problem-solving • class and group discussions • mathematical investigations • multiple representations
Create opportunities to allow students to communicate and justify their strategies and solutions	• mind mapping • low-floor, high-ceiling learning tasks • collaborative learning tasks • 'How did you do it?' • modelling • use of individual, pair and small group work
Ensure all learning experiences within the program are purposeful, developmentally appropriate, and support the long-term learning outcomes	• specific and deliberately planned activities • inclusive and differentiated tasks to suit the specific classroom context
Develop transferable skills and thinking processes through problem-solving	• real-life relatable content • student driven activity planning • research of relevant topics in social media • activities which clearly demonstrate transferable skills
Provide an appropriate level of challenge that is fair, flexible and meaningful to students	• differentiated tasks with multiple entry levels • modified programs for support and extension • enrichment and challenge • competitions

More information about the Ways of Teaching Mathematics can be found at https://k10outline.scsa.wa.edu.au/home/teaching/curriculum-browser/mathematics-v8/overview/ways-of-teaching.

Note: links to electronic resources

This sequence of lessons may utilise electronic web-based resources, such as YouTube videos. Schools are advised to install advertising blocking software prior to using online material. Additionally, teachers should be present while an electronic resource is in use and close links immediately after a resource, such as a video, has played to prevent default 'auto play' of additional videos. Where resources are referred for home study, they should be uploaded through Connect, or an equivalent system, that filters advertising content.

Ways of assessing

Fine-grained formative and diagnostic assessment strategies have been included within this teaching, learning and assessment exemplar. This way of gathering evidence is to be used by the teacher to inform the structure, pace and content of the ongoing sequence of lessons. This may occur through a range of means, including (but not limited to):

- questioning
- observing
- short quizzes (online or written)
- collection or supervision of work completed over lesson/s
- class/group work and discussion
- checklists.

It is through a variety of feedback types: self, peer, written, verbal, descriptive, evaluative, formal or informal, that students should be supported in meeting the demands of learning intentions and summative assessment tasks.

Pythagoras’ TV-rem | Response Task

This exemplar can be used to develop students’ understanding of the relationships between number, algebra, measurement and geometry. They will develop links between points on the Cartesian plane, examining the distance, midpoint and gradient of such points. In investigating the distance between two points, students will be introduced to Pythagoras’ Theorem, linking the application on the Cartesian plane to more real-life situations. Throughout the teaching and learning sequence, teachers will provide students with the opportunity to develop mathematical proficiencies within the Number and Algebra and Measurement and Geometry strands to enable students to complete the summative assessment task at the end of the sequence.

The purpose of this assessment is to give the students an opportunity to demonstrate their knowledge of Pythagoras’ Theorem and coordinate geometry. Students will demonstrate their ability to apply these skills in familiar and unfamiliar real-life applications.

If the suggested learning experiences and the relevant syllabus content for this unit have been studied, students will be well positioned to address the requirements of the assessment task to the best of their ability.

The exemplar presents a teaching and learning sequence that will enable students to understand and apply the appropriate Number and Algebra, and Measurement and Geometry to a range of everyday problems, setting them up for success when working with linear and quadratic equations in Year 10 and applying their measurement skills in more complex situations in later schooling.

Curriculum | What will be taught

Content from the Western Australian curriculum

Note: this exemplar addresses aspects of the Year 9 Mathematics content identified below.

Measurement and Geometry

- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Number and Algebra

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

Achievement Standard | What is assessed

Achievement standard

Note: areas assessed through the teaching and learning sequence presented in this exemplar are indicated in bold.

Number and Algebra

At Standard, students solve problems involving simple interest. They apply the index laws to numbers and express numbers in scientific notation. Students expand binomial expressions. **They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment**. Students sketch linear and non-linear relations.

Measurement and Geometry

Students interpret ratio and scale factors in similar figures. They explain similarity of triangles. Students recognise the connections between similarity and the trigonometric ratios. They calculate areas of shapes and the volume and surface area of right prisms and cylinders. **Students use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles.**

Statistics and Probability

Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They compare techniques for collecting data from primary and secondary sources. Students construct histograms and back-to-back stem-and-leaf plots. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Lesson 1: Review of coordinates

Western Australian curriculum content

- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point [Year 7 content]
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 1)
Learning intentions • Revise Year 7 content. • Plot coordinates on a Cartesian plane. • Write Cartesian coordinates for a point on a Cartesian plane. • Identify the four quadrants. **Focus questions** • Where might we use coordinates in everyday life? • How can you remember which axis each coordinate relates to?	**Sequence** 1. Warm-up related to Cartesian coordinates using an activity such as Cartesian coordinates figures. In this activity, students place figures of different sizes and shapes on a Cartesian plane from –10 to 10. This can be modified to support students who are not strong with negative numbers by including either all coordinates in the first quadrant or using only the first quadrant and either the second or fourth quadrant. (Appendix A.2) 2. Write a pair of coordinates on the board. Students use a whiteboard or tablet to show 1, 2, 3 or 4 to represent which quadrant the coordinates are located. Use coordinates, such as $(0, 0)$, $(0, 4)$ and $(5, 0)$, to invite critical thinking about which quadrant coordinates on the axis are (the definition states they are not in any quadrant). 3. Display a Cartesian plane on the board or projector, getting students to come up in groups of four or five to place a sticker on a given coordinate. Students write the coordinates of their sticker on the board, checking and recording their understanding as they go. Use this as a formative assessment opportunity to determine where the class is at with their understanding of plotting Cartesian coordinates.

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 1)
Formative assessment This lesson serves as an introductory lesson and, therefore, has formative assessment opportunities included at various points. Use the questioning in the whiteboard activity, the coordinate stickers and the results of the labelled coordinates to determine whether students are at or below standard. This lesson will not differentiate if any students are above the Year 9 standard; however, it can guide which students may be prepared to work above Standard in the future. • Why are the origin and the axis intercepts not considered to be part of a quadrant? • Which two quadrants have positive x-coordinates? • Which two quadrants have negative x-coordinates?	4. Label up to 26 coordinates on this Cartesian plane A through to Z, starting in the first quadrant and working around in an anti-clockwise direction. Students label each of the coordinates with the appropriate coordinate pair in their books. Ask each student to share an answer they are confident of, allowing them to pass if they aren't confident of their answers. After all answers are shared and confirmed, have students add up their tallies in their book. While they are working on consolidation, check these tallies to further determine student achievement at the start of this topic. 5. Formally define the parts of the Cartesian plane and Cartesian coordinates. Focus on the axes, the quadrants and what each part of the coordinate pair represents. Remind students that these are pairs that are linked together, so they will always need to have a set of parentheses around them, with the comma separating them from each other (i.e. x, y). 6. If students have demonstrated a collective competency in the warm-up and diagnostic activities, proceed to drawing a picture using Cartesian coordinates. Examples include: • Mr. Street's Geeky Graphs https://sites.google.com/site/mrstreetsgeekygraphs/products-services • Coordinate Picture Graphing (Worksheet Works.com) https://www.worksheetworks.com/math/geometry/graphing/coordinate-pictures.html • Graphing Worksheets for Practice (Math-Aids.com) https://www.math-aids.com/Graphing/. Encourage students to do the hardest picture that they can which interests them. Some have lots of negatives, some have lots of decimals, so assist students in choosing an appropriate resource here or allocate accordingly. 7. If students have not demonstrated a collective competency, or selected students have been identified, intervene with appropriate consolidation activities to practise plotting and identifying coordinates. This links back to a Year 7 skill, so most students should be able to demonstrate it with small interventions.

Teaching and learning intentions and assessment	Suggested learning experiences Lesson 1
	Extension opportunities • Students find a picture of their favourite character from literature or popculture. They use tracing paper or otherwise to recreate the character on a Cartesian plane, and write a set of coordinates which could be used to draw the character. • To further extend this idea, provide students with a wider grid where they need to use either decimals or fractions to accurately draw their picture and determine the coordinates. • This can be collected and utilised in later lessons for early finishers or as a brain break option.

Lesson 2: Introduction to midpoint

Western Australian curriculum content

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 2)
Learning intentions • Relate the midpoint of two Cartesian coordinates to mean or median. • Determine the horizontal and vertical midpoints and use these to determine the midpoint of a line segment. **Focus questions** • When determining the middle of two things, what are we doing mathematically? • How far is the midpoint from each point? • Is the midpoint of a line segment always between the two points?	**Sequence** 1. Start lesson with a review of determining the mean, mode and median of small data sets. This is not specifically used in this lesson, but the idea of using the mean or median to determine the centre of two points is essential to developing understanding. 2. Provide students with examples of pairs of objects; for example, two sticks of different lengths, two bags of sand with different masses, two devices with different charges, two shoes of different sizes. Each pair should have the same object with different measurements; for example, two cups of water, one having 100 mL and the other 160 mL. Ask students to work out the dimensions of a third object (using the same units) which is exactly in the middle of the two provided. In the given example, this would be a cup with 130 mL. If possible, provide students with scales, tape measures, rules and beakers which allow them to actively measure the objects. Students create a table in their books which has one column for the each object, one column for the total of the two objects and one column for the middle object. 3. Print four or five number lines for each student. It is suggested that the number lines span from –10 to 10. Provide students with two even counting numbers. Model the process of folding the number line to make the points meet up. Ask them what the location of the fold represents. Practise this method using a combination of one odd and one even positive, both negative, pairs that cross 0, and even numbers which are non-integers. Question students about how they could determine the value of this point without folding the paper every time. Prompt them to move towards the idea of adding the numbers up and dividing them by two. (Appendix A.3)

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 2)
Formative assessment Use 'How did you do it?' questioning to identify how students are thinking about addition and subtraction. If students are struggling with the concepts, help to model them by using a number line or other appropriate strategy.	4. Pick two students in the class at random, have them stand up and ask a third student to try and stand so they are in the middle of both students. Ask the rest of the class to comment on the accuracy of this prediction and have them instruct where to move to, if required. Practise this a few times, with students in different orientations within the room. 5. Provide students with a Cartesian plane with a series of Cartesian coordinates plotted and labelled from A to S. Students visually locate the midpoint of at least 10 sets of coordinates that they are comfortable to determine and fill out a retrieval chart. It is intended that they will develop the rule as they work. Challenge them to be able to work out the middle of any two Cartesian coordinates. (Appendix A.4) 6. Once students think they can determine the midpoint of any two Cartesian coordinates, work with coordinates of real numbers involving decimals, fractions, very large and very small numbers, as well as some irrational numbers. This should be offered as extension to students who have demonstrated they are working above the expected Standard. 7. Consolidate learning through an appropriate learning activity. • Use a deck of cards to practise. Students can work in pairs to create coordinates using a deck of cards. This could be easily modelled using the black cards to be positive and the red cards to be negative coordinates. Each student creates a Cartesian coordinate using two cards and then they both calculate the midpoint using these cards. Students show working in their workbook. • Reflect on students' needs and choose an appropriate web resource for their ability. ▪ Midpoint Formula Worksheets (MATH Worksheets 4 Kids) https://www.mathworksheets4kids.com/midpoint-formula.php ▪ Midpoint of a Line Segment Worksheets (Easy Teacher Worksheets) https://www.easyteacherworksheets.com/math/geometry-midpoints.html ▪ Free Pre-Algebra Worksheets | Plane Figures | The midpoint formula (Kuta Software) https://www.kutasoftware.com/freeipa.html -> The midpoint formula

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 2)
	▪ Videos and Worksheets | Coordinates: midpoint of a line (Corbett Maths) https://corbettmaths.com/contents/ ▪ Midpoint of a Line Segment Worksheets (Tutoring Hour) https://www.tutoringhour.com/worksheets/midpoint-formula/line-segment/. 8. Pose a problem on an exit ticket for each student. If students have demonstrated understanding, problems could focus on determining the end point if the midpoint and the other end point are known, otherwise problems focus on a calculation of the midpoint. **Extension opportunities** • Students work out the coordinates that split a line segment into three or four even parts. • Students determine the end point when given a midpoint and other end points (same as the exit ticket). • Students work out the coordinates of the end point when given one end point and a quarter point. Extend this to other simple fractions.

Lesson 3: Application of midpoint

Western Australian curriculum content

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment	Suggested learning experiences Lesson 3
Learning intentions • Consolidate and apply working with midpoints in unfamiliar contexts. • Relate the midpoint of a line segment to the midpoint of a plane using algebraic and geometric approaches. **Focus questions** • What do you notice when you join the midpoints of the edges of a triangle? • What do you notice when you join the midpoints of the adjoining edges of a quadrilateral? • How can you use a midpoint to	**Sequence** 1. Introduce lesson with a practice question to determine the end point given the midpoint and the other end point. Ask students whether it matters which point is the end and which point is the midpoint? Will it matter if we change which is which? 2. In groups of four to six, students complete an activity exploring the centre of gravity of triangles. This activity is an open-ended exploration involving the relationship between geometry, space, the Cartesian plane and measurement. (Appendix A.5) Once students use cut-outs of triangles to determine the centre of gravity, they will explore why these occur in the specific locations they do and develop a rule algebraically and geometrically. For the teacher's reference, the centre of gravity of a two-dimensional shape occurs at the mean of the $x-$ and $y-$ coordinates of each of its vertices.

Teaching and learning intentions and assessment	Suggested learning experiences

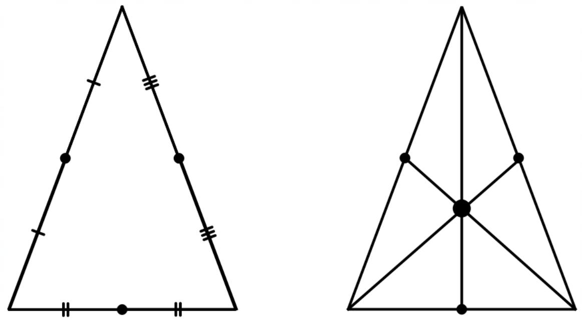
help you to determine the centre of gravity of these shapes?

- Can you draw a shape where the midpoint is not on the shape?

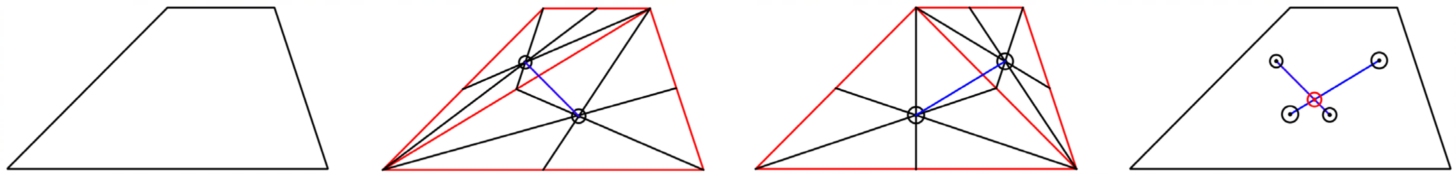
Formative assessment

Use the open nature of this activity to identify whether students are able determine the midpoint of a line segment graphically or algebraically. They should be able to use the information on the graphs to locate the coordinates of the vertices, then use these to calculate the midpoint. Students who are above the expected Standard will be able to work in this context to determine the centre of gravity. Students who are working well above the Standard will be able to apply multiple steps to determine the centre of gravity of triangles and quadrilaterals.

Geometrically, it differs for each shape, but for a triangle it occurs where the line segment connecting to the opposing vertex meets the same line segment connecting the other two midpoint-vertex lines. This is shown on the diagram below.

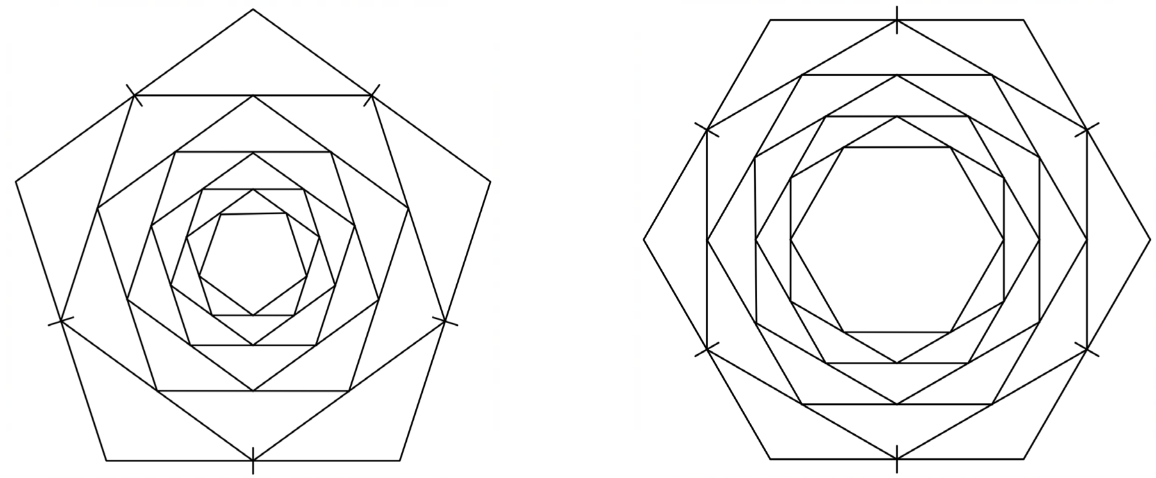


3. As a challenge, students can attempt to determine the centre of gravity of some simple quadrilaterals and more complicated polygons. The algebraic determination of the centre of gravity still stands; however, geometrically it becomes more complicated with increasing numbers of sides. For instance, in quadrilaterals, the shape must be split by joining two opposite vertices to form two triangles. The centres of gravity of these triangles is determined using the previous method and connected with a line segment. The quadrilateral is then split by the other diagonal and the process repeated. Where these two line segments intersect is the centre of gravity for this shape. This is shown below.



4. Once students have explored the centre of gravity of these shapes, they can move on to an activity looking at the shapes formed when adjacent midpoints are connected. This activity looks at connecting the midpoints of polygons and exploring the shapes that are created using this construction. Students start with a range of triangles, then move to

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 3)
	regular and irregular quadrilaterals. Students write a short report about their findings with these shapes. For the teacher's reference, triangles always make congruent shapes, because of the parallel lines formed. All quadrilaterals form parallelograms, for a similar reason. (Appendix A.6) This activity should go for all of Lesson 3 due to the open nature of the questions. Students can delve as deep as they are capable of, moving into pentagons, hexagons and convex quadrilaterals. **Extension opportunities** • Examine the nets created by folding the line segments connecting midpoints of triangles. • Examine the area formed by connecting the line segments connecting midpoints of triangles and quadrilaterals.

Teaching and learning intentions and assessment	Suggested learning experiences
	• Examine the patterns formed by connecting the midpoints of a regular pentagon and hexagon. Use this to create a mindful drawing activity and, if students have covered similar triangles this year, look at the triangles formed in the repeated connection of the midpoints of these shapes. A regular pentagon and hexagon have been included in this resource (Appendix A.7) 

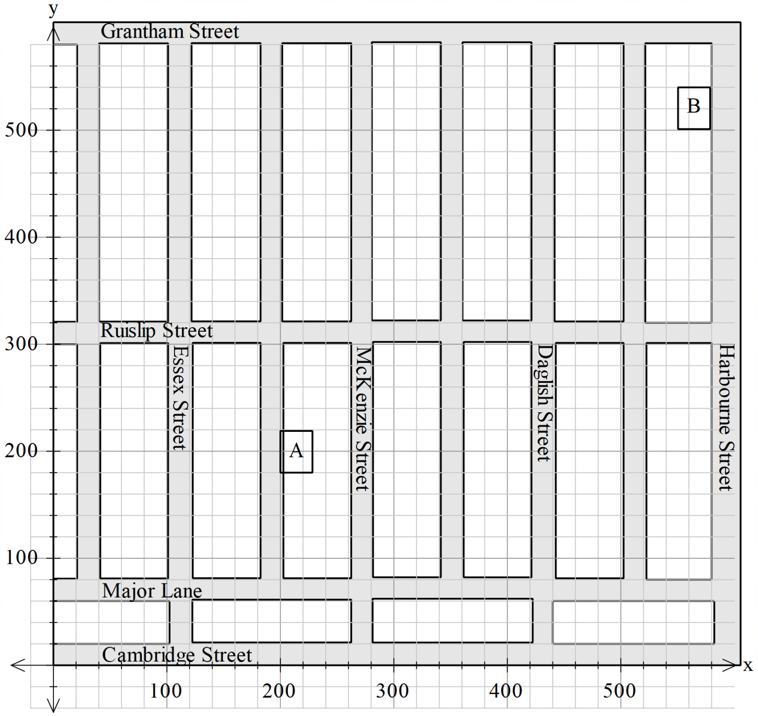
Lesson 3

Lesson 4: Introduction to distance between two points

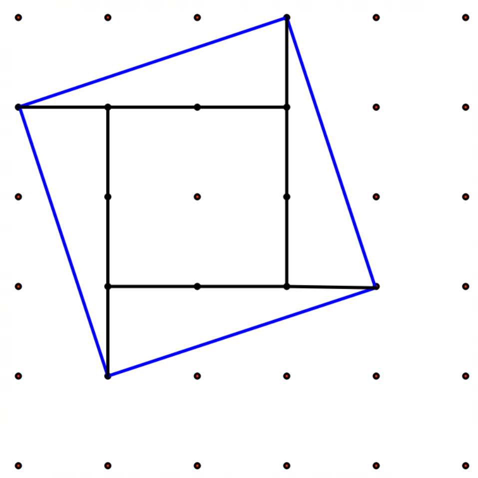
Western Australian curriculum content

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 4)
Learning intentions • Determine the length of a line segment by measuring and inspecting. • Develop a formula to determine the distance between two points in a vertical or horizontal line. **Focus questions** • What shape is made when looking at the horizontal and vertical distances between two points? • What type of triangle is this? • Will all points with horizontal and vertical legs that add to the same length have the same distance	**Sequence** 1. Warm up with squares and square roots mental maths questions. 2. Locate an appropriate map of either the Perth CBD or a district relevant for your school context. Ensure that this map has vertical and horizontal streets. Place grid over the map so the map occurs in the first quadrant only. An example of this is shown below.

Teaching and learning intentions and assessment	Suggested learning experiences
between them? Provide an example. **Formative assessment** Observe student progress during the Geoboards activity. Ask questions about which side is the longest of the three making up the triangle, and whether this is the same for all right-angled triangles. During Learning experience 4, allow students the opportunity to demonstrate their problem-solving skills first, prior to showing them how the slanted squares work. Ask whether there is a relationship between the size of the squares and the vertical and horizontal lengths of the line segments.	Use this example to discuss the logistics of getting from one location to another and back again. Determine the total distance travelled using the streets. Ask students if they think this distance would be longer or shorter in a straight line. Ask them to estimate what they think the distance would be and to provide their answer as a range of values.  Use an appropriate measure (either using scale or otherwise) to determine the distance between the two points. Note: scale is not formally taught or assessed in this exemplar; however, this provides a good opportunity to amend the sequence and add an activity involving scale, as appropriate to a class/students.

Lesson 4

Teaching and learning intentions and assessment	Suggested learning experiences
	3. Use a tool such as GeoBoards for students to practise comparing the distances between two points using the horizontal and vertical distances and the straight line measure. This would be easier to measure physically using a grid system or even a blank page, as long as the students are able to create a right-angled triangle. • Geoboard by The Math Learning Centre (The Math Learning Centre) https://apps.mathlearningcenter.org/geoboard/. 4. Give students a 6 x 6 dotted grid, and ask them to draw squares with as many different areas as possible. Go through the five which align to the grid (1 cm^2, 4 cm^2, 9 cm^2, 16 cm^2, 25 cm^2) as a class, then ask if there are any more squares that are possible. This should lead to students using angled squares, such as shown below. Help students determine the area of these squares, by splitting them into component squares and triangles. The example below shows a square made up of line segments which have a vertical distance of one unit and a horizontal distance of three units (or the opposite). The area of this square is determined by looking at the individual shapes (4 triangles and the square). Ask students to determine the length of the side of the square using the formula $A = l^2$ or $l = \sqrt{A}$. 

Lesson 4

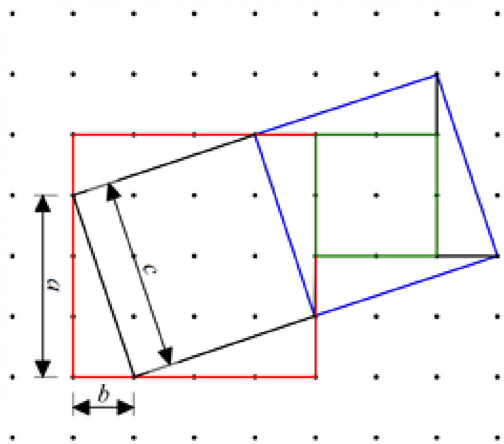
Teaching and learning intentions and assessment	Suggested learning experiences

Lesson 4

5. Model determining the distance between two points using a set of Cartesian coordinates, first measuring the distance between the two points to two decimal places. Students compare this to the square root of the area of the square this line segment forms and comment on the accuracy of both methods.
6. Extend activity 5, having students measure the distance to the midpoint and then calculate it more formally by making two smaller squares. This leads to an opportunity to ask questions about why the area of the square of the whole line is four times larger than the square formed by the line segment to the midpoint.
7. Provide opportunities for consolidation as required.
   Note: students have not formally learned the distance formula at this point, so will need to use the squaring approach in any consolidation activities.

Extension opportunities

- If it has been covered previously, this model also encompasses the algebraic expansions of $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$ as shown in the diagram below.



Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 4)
	The line segment used is a units horizontally and b unit vertically. The algebraic expansion shows that the square formed by adding these two lengths together is $(a + b)^2$ which is the square outlined in red. This square is made up of 4 triangles with a total area of $2 \times a \times b$ and the black square in the centre, with an area of c^2. The total area of these components is $2ab + c^2$ and the expansion of $(a + b)^2$ is $a^2 + 2ab + b^2$. This means that c^2 must be exactly the same as $a^2 + b^2$. This also holds true for the smaller green square. The algebraic expansion shows that the square formed by subtracting the smaller length from the large length is $(a - b)^2$. The blue square is made up of the same 4 triangles, with the area of $2 \times a \times b$ and the green square in the centre with an area of $(a - b)^2$. The relationship between these components is that subtracting the triangles from the blue square leaves the green square. Algebraically, this is $c^2 - 2ab = a^2 - 2ab + b^2$ which again shows us that c^2 must be exactly the same as $a^2 + b^2$. • Provide examples of questions with multiple adjoining line segments for students to determine the overall horizontal and vertical distance from the start point to the end point. They compare this to the total horizontal and vertical distances travelled in the examples.

Lesson 5: Introduction to Pythagoras’ Theorem

Western Australian curriculum content

- Investigate Pythagoras’ Theorem and its application to solving simple problems involving right-angled triangles
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software

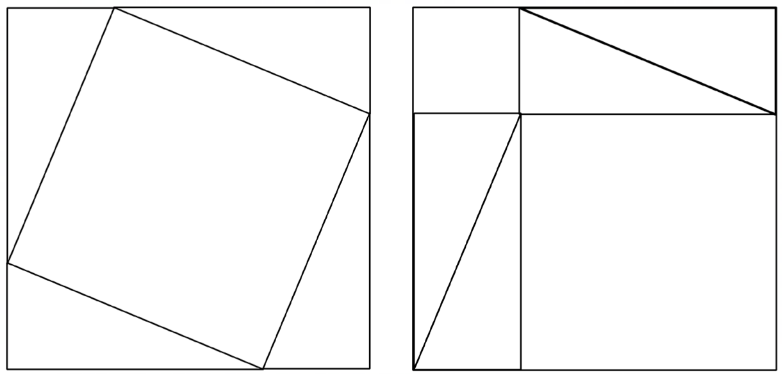
<table>
<tr><th>Teaching and learning intentions and assessment</th><th>Suggested learning experiences
Lesson 5</th></tr>
<tr><td>
Learning intentions

• Develop understanding of Pythagoras’ Theorem through investigation and experimentation with shapes and measurements.
• State Pythagoras’ Theorem.
• Use Pythagoras’ Theorem to determine the length of the hypotenuse and of a shorter side.

Focus questions

During Learning experience 2:
• What did you notice about the area of the two squares your pair made?
• What did they have in common?
</td><td>
Sequence

1. Warm up with square and square root practice questions.

2. Provide students with the template from Appendix A.8. Students cut out the triangle templates and put them back together to make a complete square. This should lead to students making one of the shapes below. If appropriate, create a template to reflect both of these options and have adjacent students work on different templates. This can be modelled electronically using an applet such as:
• Pythagoras Proof [GGB] (Interactive Maths) https://www.interactive-maths.com/pythagoras-proof-ggb.html.

</td></tr>
</table>

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 5)
What did they have that was different? • What can you say about the area of the larger square and the area of the two smaller squares? • How do these shapes relate to the right-angled triangles present? How can we speak about their lengths and areas with respect to these triangles? During Learning experience 3: • What do you notice about the area of all of the squares? • Are there any triangles you can make where the area of all of the squares are square numbers? What must the lengths of the sides be in this case? • Is there a point where the larger area is the same as either of the other areas?	3. Show the relationship between the squares on the side lengths using a website, such as: • Pythagoras Theorem [GGB] (Interactive Maths) https://www.interactive-maths.com/pythagoras-theorem-ggb.html. This can be used to prompt students to think about what the formula for Pythagoras' Theorem might be. 4. Provide students with a range of right-angled triangles. (Appendix A.10) Students complete the table in the same appendix. They will measure the lengths of each of the sides, then square the lengths and sum a^2 and b^2. 5. Introduce the relationship $a^2 + b^2 = c^2$ and demonstrate with examples involving labelled triangles to determine the length of the hypotenuse and the shorter sides. Use guided practice to develop skills and questioning and observation to determine whether students are able to work with the rule. 6. Students determine if a series of three numbers could represent a right-angled triangle. Prepare examples of three measurements which do or do not fit the rule. Give students 30–60 seconds per question (depending on their understanding and the results of each question) and show a tick or a cross on their whiteboards or tablets. 7. Opportunities for consolidation Pythagoras with cards: in pairs, students draw one card each from a deck of cards with the picture cards removed. They use these numbers to draw and label a triangle, using their cards as the shorter lengths to determine the length of hypotenuse. This can be extended to include two cards which represent a two-digit number or a decimal number. Students can choose to use the larger card as the hypotenuse and work out the length of a smaller side. Reflect on the learning needs of the students and choose an appropriate web resource, such as: • Pythagoras Theorem Activities (Interactive Maths) https://www.interactive-maths.com/pythagoras-theorem-activities.html • Pythagoras Theorem Questions (Math-Salamanders)

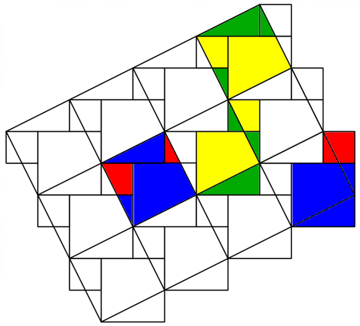
Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 5)
During Learning experience 5: • Is it possible to use negative numbers for the side lengths? What happens to them in the formula? • Is there any case where the side lengths are larger than the hypotenuse? • Are there any right-angled triangles which can be made up of whole number measurements only? **Formative assessment** This lesson has multiple opportunities for questioning, observation and student reflection to identify the achievement of students. Each activity has opportunities for the teacher to ask key questions to gauge understanding before moving to the next activity.	https://www.math-salamanders.com/pythagoras-theorem-questions.html • Pythagoras' Theorem: find the length of the hypotenuse (IXL Learning) https://au.ixl.com/math/year-9/pythagoras-theorem-find-the-length-of-the-hypotenuse • Pythagoras' Theorem (MathsIsFun) https://www.mathsisfun.com/pythagoras.html Scroll down to questions • Pythagorean Theorem 1 (Khan Academy) https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/cc-8th-pythagorean-theorem/e/pythagorean_theorem_1. 8. Collect an exit ticket which has the answer to one of the following questions. (Appendix A.9) Students select the hardest question they think they can solve. • What is the relationship between the sides in a right-angled triangle? • Which side is the longest? • A triangle has shorter sides of 3 cm and 4 cm. What is the length of the hypotenuse? • A triangle has one short side that is 6 cm and the hypotenuse is 10 cm. What is the length of the other short side?

Teaching and learning intentions and assessment	Suggested learning experiences
The exit tickets will give an indication of where the students are at individually. Use these exit tickets to determine the pitch of the entry activities in the next lesson. If more time is required to consolidate skills, build this into the teaching and learning sequence.	**Extension opportunities**

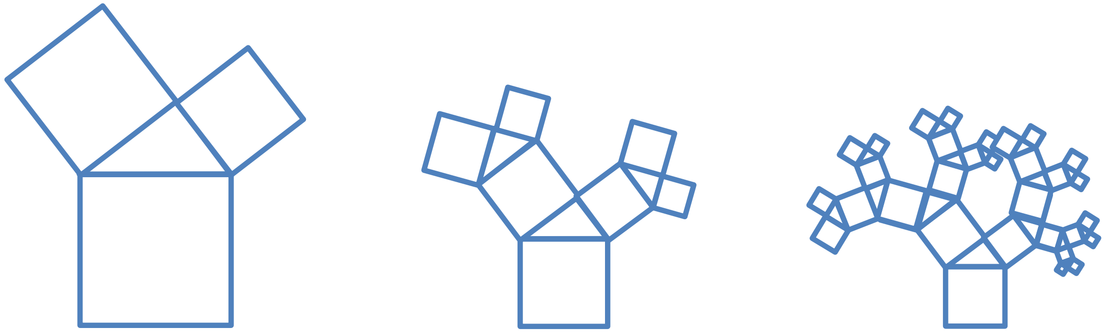
Lesson 5

Extension opportunities

- Students create a repeating pattern, which models Pythagoras' Theorem, such as that shown below. Provide them with graph paper and see what patterns they can create which model Pythagoras' Theorem. This is also called Pythagorean tiling.



- Students construct a Pythagorean tree. This involves starting with the basic visual representation of Pythagoras' Theorem. Each of the squares on the smaller side are then used to represent the square of the hypotenuse of a smaller triangle as shown below. Students repeat the pattern and continue to make a Pythagorean tree. Challenge them to determine the dimensions of each shape and the total area of the tree after adding each part.

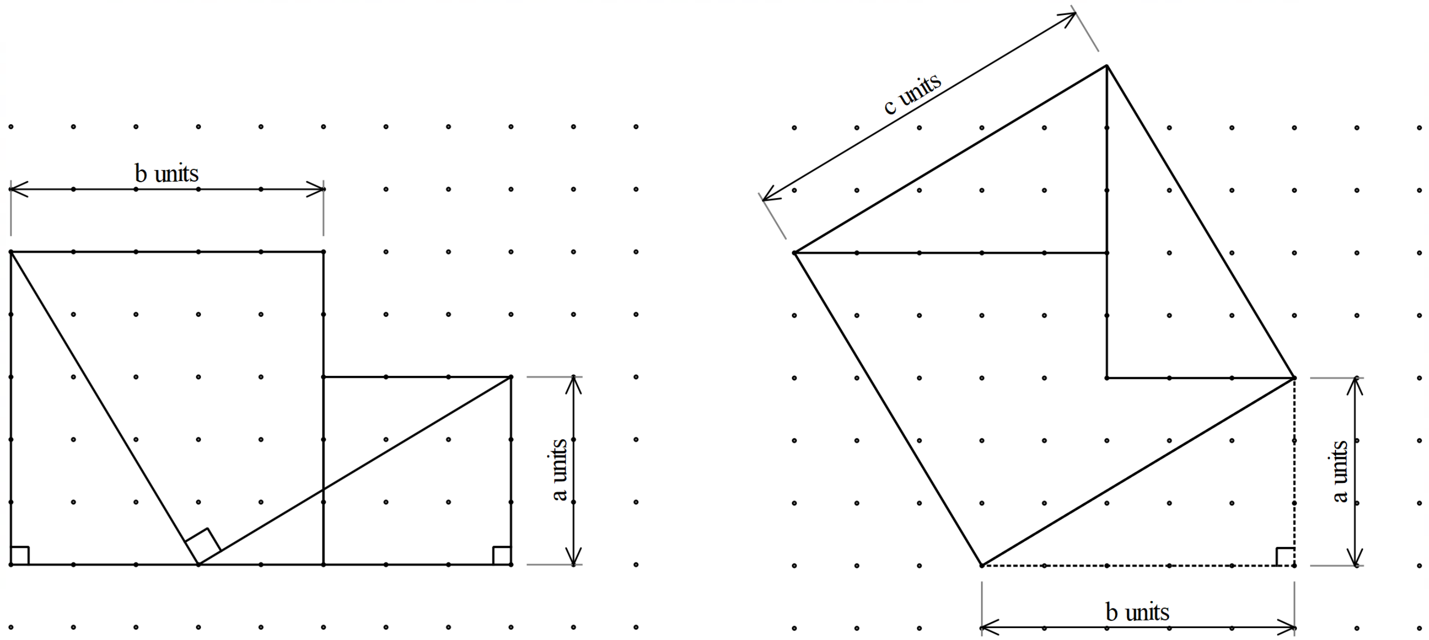


Lesson 6: Applications of Pythagoras' Theorem

Western Australian curriculum content

- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 6)
Learning intentions • State Pythagoras' Theorem. • Use Pythagoras' Theorem to solve the length of the hypotenuse or the shorter sides in a right-angled triangle. • Draw a diagram from a worded problem involving right-angled triangles, labelling it appropriately. • Extract the important information from an authentic context and represent it as a diagram. **Focus questions** • What is the difference in the operations required to find the	**Sequence** This sequenced lesson will be looking at working with, consolidating and expanding students' knowledge of Pythagoras' Theorem. If time is available, use a resource such as the Lunch Lap, which is scheduled for three lessons, to enrich and extend students: • Geometry: Lunch Lap (reSolve) https://resolve.edu.au/geometry-lunch-lap-trial. 1. As an introduction to Pythagoras as a historical figure, show the video: • Mystery man Pythagoras meets his match (ABC Education) https://education.abc.net.au/home#!/media/1003922/. 2. Students find one fact they like about Pythagoras and write it in marker to form a word wall about Pythagoras. A suitable resource is: • 50 Surprising Pythagoras facts you never knew (facts.com) https://facts.net/history/people/pythagoras-facts/. 3. Provide students with a 10 x 10 cm square of graph paper. Students draw two different sized squares next to each other, where each length is a whole number. On these squares, they draw a line segment from the top right corner of the smaller square to the bottom edge of the larger square, so it is the length of the larger square. Draw a line from the point

Teaching and learning intentions and assessment	Suggested learning experiences
hypotenuse compared to finding a short side when using Pythagoras' Theorem in a right-angled triangle? • Where might you see Pythagoras' Theorem used in everyday life? • Computer monitors are measured by their hypotenuse. Can you explain why knowing a computer monitor is 60 cm doesn't help to show exactly what the shape of the monitor is? • A Pythagorean triad is when three whole numbers make up the measurements of a right-angled triangle. The simplest is 3, 4 and 5. How can we use this knowledge to determine the hypotenuse of a triangle with shorter sides of 6 and 8? List three other triangles with this same side length ratio.	where this line segment intersects the bottom edge to the top left corner of the larger square. Students cut out the two resulting triangles and place them to make a single square. They summarise what they have observed in their own words and share using a think-pair-share. A model of this process is demonstrated below: 

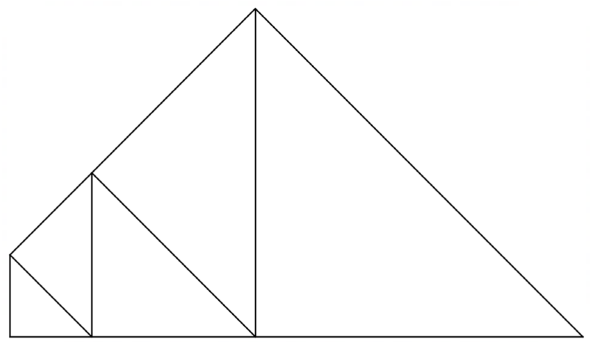
Lesson 6

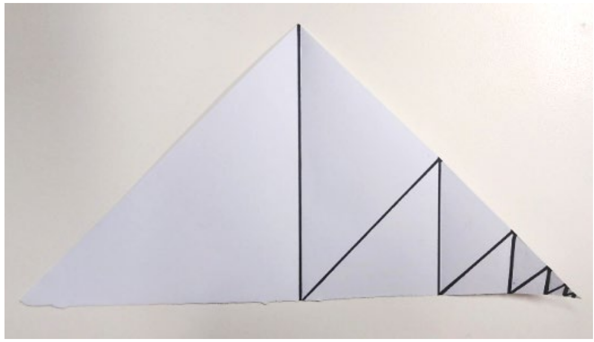
Teaching and learning intentions and assessment	Suggested learning experiences Lesson 6
Formative assessment During the course of this lesson, check on students' ability to calculate any unknown side length in a right-angled triangle. Start with students who have been identified previously as at risk of not being at the expected Standard. These students will come up to work one-on-one with the teacher to solve one problem with a diagram and one simple word problem. Record their progress in a checklist to indicate their achievement.	4. Warm up using Pythagoras' Theorem by having a guided practice to find the hypotenuse and the short side in a triangle. An alternative electronic resource can be found at: • Pythagoras' Theorem Exercise (Transum) https://www.transum.org/software/SW/Starter_of_the_day/Students/pythagoras.asp. 5. Question students about the use of Pythagoras' Theorem and where it could be seen or used in everyday life. Choose an appropriate response and use this to build a question. Some everyday uses include: • in the construction of roofs and housing • to determine a total distance walked (looking at compass direction movement) or component parts (how far north if walk is 500 m in total and is 100 m south of starting point) • to determine the length of a ladder required to reach a window • to find the shortest distance between two points. Use an appropriate context to develop problem-solving skills, such as drawing a diagram, estimating, breaking the problem down into smaller parts, acting out the problem or other skills appropriate to the classroom context. 6. Look at the case of 3, 4, 5 making a right-angled triangle and ask students to research other whole-number right-angled triangles. Look at testing multiples of this, such as 6, 8, 10 or even 0.3, 0.4 and 0.5. Each student tries to write their own unique Pythagorean triad. Create a word wall of these. 7. Reflect on the learning needs of the students in the class and choose an appropriate web resource for consolidation, such as: • Pythagorean theorem word problems (Superprof) https://www.superprof.co.uk/resources/academic/maths/geometry/plane/pythagorean-theorem-word-problems.html • Pythagoras' Theorem word problems (IXL Learning)

Teaching and learning intentions and assessment	Suggested learning experiences
	https://au.ixl.com/math/year-9/pythagoras-theorem-word-problems

- Pythagorean Theorem word problems basic (Khan Academy) https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/pythagorean-theorem-application/e/pythagorean-theorem-word-problems--basic
- Worksheet – Worded Pythagoras Theorem questions (StudyMaths.co.uk) https://studymaths.co.uk/workout.php?workoutID=16
- Word problems on Pythagorean Theorem (Math-Only-Math) https://www.math-only-math.com/word-problems-on-pythagorean-theorem.html
- 48 Pythagorean theorem worksheet with answers (TemplateLAB) https://templatelab.com/pythagorean-theorem-worksheet/.

Extension opportunity





- Students draw a right isosceles triangle and then use this to draw continued right isosceles triangles. Invite students to investigate the patterns in the dimensions and the area of the changing triangles. Plot these on a Cartesian plane, with

Teaching and learning intentions and assessment	Suggested learning experiences
	the x-axis as the number of triangles and the y-axis as the area of the largest triangle. This can be replicated by folding a piece of paper. Square a rectangular piece of paper and split it into one triangle between two. Invite students to determine the length of the hypotenuse using the measurements of the piece of paper (21 cm x 29.7 cm). Create a competition to see who can fold it the smallest and determine the dimensions of the triangle. Students work out the ratio of the smallest triangle to the largest triangle. What percentage of the area of the largest triangle is the smallest triangle?

Lesson 7: Formative assessment

Western Australian curriculum content

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 7)
Learning intentions Identify progress in the topic by conducting formative assessment and reflecting upon students' own learning. **Formative assessment** The checklist in Appendix B will provide a visual indication of whether students are at, above or below the expected Standard. Use this to guide the teaching and learning sequence.	**Sequence** 1. This lesson involves a formative assessment which looks at applying Pythagoras' Theorem using shapes other than squares on the individual edges of a right-angled triangle to test if the relationship can be applied in other situations. (Appendix B) **Extension opportunities** If any students finish early, ask them to look into other shapes which might fit the rule. These could include pentagons, equilateral triangles, scalene triangles, other rectangles and even irregular shapes. Pose the following questions. • Why do these shapes and some other shapes not work? Can you prove why the semicircle did work? • Would a rectangle formed on the side length with a width of 1 cm work? Why/why not?

Lesson 8: Gradient of a line segment

Western Australian curriculum content

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 8)
Learning intentions • Determine the vertical and horizontal lengths of a line segment. • Use the horizontal and vertical lengths of a line segment to calculate the gradient of a line. • Identify if a gradient is positive or negative by inspection. • Identify if the magnitude of a gradient is greater than or less than one by inspection. • Calculate the gradient of a line segment between two points on the Cartesian plane.	**Sequence** 1. Introduce lesson with a round of Simon Says using gradients. Students make the gradient of a straight line using their arms as either positive, negative, zero or undefined (or infinite). It may help to have a Cartesian plane displayed on the board so students can visualise the arm movements. 2. Students work through Appendix A.11 on ski resorts, applying their knowledge of the Cartesian plane and coordinate geometry to determine the gradient of slopes. This is a group activity, where students will first rank the ski slopes from easiest to most difficult. After this, they calculate the gradient of the slopes, given pairs of Cartesian coordinates. Finally, they create their own map of ski runs for an imaginary resort. During this activity there will be opportunities to observe each group practise the skills to determine the gradient of a line segment. It is also scaffolded as part of the activity, so students should develop these skills as they go. Note: this activity may require more than one lesson for students to develop the understanding to reach the Standard. Teacher determines the time required to deliver the sequence of teaching and learning found in Appendix A.11.

Teaching and learning intentions and assessment	Suggested learning experiences
Focus questions • If there is a ski lift and a ski run on the exact same slope, what is the same and what is different about the travel occurring? • Which is going up (positive) and which is going down (negative)? • What is the slope of a flat surface? • What is the slope of a vertical surface? • If the vertical length of a slope is halved, what is the impact on the gradient of the slope? • If the vertical length of a slope is doubled, what is the impact of the gradient of the slope? **Formative assessment** This activity is student driven, so there are many opportunities for the teacher to intervene as appropriate. Use questioning and observation of individuals and groups during the	

Lesson 8

Teaching and learning intentions and assessment	Suggested learning experiences
activity to determine their level of understanding. Use knowledge of the classroom context to determine the level of support and the length of time required to complete the activity.	

Lesson 9: The distance formula

Western Australian curriculum content

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles
- Solve problems using ratio and scale factors in similar figures [Prior learning]
- Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems [Prior learning]

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 9)
Learning intentions • Use Pythagoras' Theorem to derive the distance formula. • Apply the distance formula individually by using the horizontal and vertical components of a line segment and using the coordinates. • Use a scale factor to use coordinates on a map to apply to measurements in real life.	**Sequence** 1. Warm-up determining the gradient of line segments on a Cartesian plane. 2. Review Pythagoras' Theorem with two triangles drawn and a worded problem which includes decimals. 3. Display a set of Cartesian coordinates which show the location of several Australian capital cities. (Appendix A.12) Students determine the coordinates of each capital city. 4. As a class, look at the coordinates of Perth $(-32, -12.5)$ and Melbourne $(20, -24.5)$. At this stage, students should be able to connect these cities by a straight line and then draw the horizontal and vertical distances to create a right-angled triangle. Ask students how they can calculate the horizontal and vertical distances. If students are struggling to come up with an answer, ask which operation, addition, subtraction, multiplication or division gives them the distance. Students test their decision with two different cities. Students should have come up with $(x_2 - x_1)$ for the horizontal distance and $(y_2 - y_1)$ for the vertical distance. If students have not discovered this, explain it to them.

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 9)
Focus questions • How does the distance between two points on the Cartesian plane relate to Pythagoras' Theorem? • How can we use Pythagoras' Theorem to calculate this distance? • Why can't we know the precise location of a coordinate if we have the length of the line segment and the horizontal or vertical change only? • The distance between Perth and Melbourne is 2727 km, however, an estimate from this graph resulted in 2668.33 km. What are the potential sources of error in this calculation and how could they be minimised?	Students use Pythagoras' Theorem to determine the distance between Perth and Melbourne (53.367 units). They then check what the distance between these two cities is using the web (2727 km). Ask why these two numbers are different. Students should recognise that there is a scale factor required to determine the exact answer. In this map, 1 unit represents 50 km. Multiply the number of units by 50 to get the actual distance from this map (2668.33 km). Students work out what the percentage difference is. Discuss why there might be a difference in the values and where this source of error could come from. Choose two new cities such as Darwin $(-4, 37.5)$ and Brisbane $(37.5, -2)$. Ask if there is a way we can go straight from the coordinates to the distance between them in one calculator operation. Students write down step by step what they would do, then try and recreate this as one singular operation. 5. Introduce the distance formula, $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and have students apply this to their calculations for Perth to Melbourne and Darwin to Brisbane. Ask students if the order of the points matters. Follow this up by asking what happens to negative numbers when they are squared. Remind them of the importance of ensuring that the coordinates are represented as (x_1, y_1) and (x_2, y_2) as this impacts gradient greatly. 6. Students calculate the distance between each city, and record their answers in the table the end of Appendix A.12. **Extension opportunity** • Students look at splitting a state or country into regions whose capital city is the closest capital city geographically. A good example of this is the far northern region of Western Australia, to which Darwin is five or six times closer than Perth. Ask students: what might state borders look like if every part within a state was closest to its capital city? An example of this is areas of far north Western Australia which are geographically closer to Darwin than Perth. Use your knowledge of

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 9)
Formative assessment Students are using multiple strands to access the content of this lesson. As such, it is important to provide appropriate scaffolding to students where required. If students require this scaffolding, use this opportunity to work with them one-to-one to diagnose where the misconceptions have occurred in the prior learning. Use this information to formulate the appropriate revision required during Lesson 11.	midpoints to explore this idea. There is a video series on Khan Academy which shows how midpoint and gradient are used to construct naturally occurring patterns, such as the spots on a giraffe, the patterns bubbles make when they combine and the cracks which appear in mud. Animators from Pixar talk about how they use this technique in the video found at the Khan Academy website: ▪ *Voronoi Partition* [Video] https://www.khanacademy.org/computing/pixar/pattern/dino/v/patterns2_new. This idea can be compounded using a game, such as that found at Git Hub: ▪ Voronoi diagram area game http://cfbrasz.github.io/VoronoiColoring.html.

Lesson 10: Bringing it all together

Western Australian curriculum content

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 10)
Learning intentions • Identify applications of Pythagoras' Theorem in real-life contexts. • Use Pythagoras' Theorem, distance formula, midpoint and gradient in conjunction in a single context. **Focus questions** • What are the possible errors that someone could make when calculating the distance, midpoint or gradient of a line segment? • How could you check to see	**Sequence** 1. Show the students a video of a horse and rider completing a dressage routine. For example: • Charlotte Dujadin's World Record Breaking Freestyle – Reem Acra FEI World Cup™ Dressage 2013/14 | FEI https://www.youtube.com/watch?v=tcIzsC7_Has. Set the following problem for students. A local dressage club uses ropes of a known length to set up the 60 m by 20 m rectangular area used. The club has ropes which are 12 m, 15 m, 20 m, 60 m, 80 m and 100 m. Explain how the club can use these ropes to set up the area without measuring anything, making sure the angles are exactly 90 degrees. For show jumping, the arena will either be 60 m by 40 m or 60 m by 100 m. What ropes would they need as a minimum to set up either of these two arenas? 2. Allocate students to mixed-ability groups of four. Provide each student with a copy of the template in Appendix A.14. Students draw two pairs of coordinates of their choosing and pass their page to the person on their left. This person

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 10)
whether someone had made one of these errors? **Formative assessment** Use the exit ticket to determine students' confidence in working with each of the specific topics covered in this unit. If appropriate, provide an extra lesson if there is common content which is poorly rated by a majority of the students.	calculates the horizontal and vertical distance, the length and gradient of the line segment and the coordinates of the midpoint in their book. They then add two pairs of coordinates to the next diagram and pass to the left again. This person completes the same calculations for both sets of coordinates. They repeat this process until all four planes have a set of coordinates on them. At the completion of this process, students compare their solutions and come to an agreement on where any mistakes may have been made. 3. On the reverse of Appendix A.14, students draw two triangles, one with an unknown length on the hypotenuse and one with an unknown length on a short side. They write a worded problem to be solved using Pythagoras' Theorem. Students are responsible for making sure they can solve their own problems before passing them to a partner. Each student solves their problems on the reverse of Appendix A.14 and passes the questions to the next person. If students do not understand the working, model it for the class. 4. In preparation for the upcoming revision session, students rank the skills they have learned from those they feel most confident using to the least confident. This can be done through an online survey or in their books. Use this opportunity to identify the common concerns of students to direct the focus of the revision lesson. As an exit ticket, give students a template to collect their feedback, such as the one below.

Teaching and learning intentions and assessment	Suggested learning experiences

Lesson 10

Topic	Rank
Midpoint	
Distance between points	
Gradient	
Pythagoras	

Notes:

5. Give students an opportunity to start their revision by consolidating with an appropriate resource.

Extension opportunities

- Time permitting, students go out of the classroom and find right-angled triangles or squares and rectangles they can split into right-angled triangles. They take a photo using a tablet or other device, then measure the legs or hypotenuse and one side to then calculate the other. This can be compared to the actual measurement if, available.
- Work with small groups of students who have demonstrated they are above the expected Standard on how to operate with surds, when working with squares and square roots. Demonstrate why working with an answer such as $\sqrt{2}$ is more accurate than 1.41 in continuing to do calculations.

Lesson 11: Review for summative assessment

Western Australian curriculum content

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 11)
Learning intentions • Review understanding of all content covered in this topic. • Prepare for upcoming summative assessment. **Formative assessment** This lesson is very flexible with respect to the specific classroom context. If the activity needs to be modified to be more scaffolded or more difficult, this is at the discretion of the teacher. Students should be provided with a range of problems, with the solutions,	**Sequence** 1. Use the exit ticket information from the last lesson to guide the starting activity for this lesson. If there is one activity that students feel on the whole they need to practise more, use a warm-up involving skills related to this. 2. Arrange students into eight groups. These can be mixed-ability groups using the exit tickets from last lesson, or can be groups who have identified similar areas in need of improvement. Set up 16 sets of stations around the room. There will be eight different activities, two of each set, so everyone can be working from the basic understanding of a skill to the application of the skill. The eight stations will have a range of questions on the following topics:

Using Pythagoras' Theorem in right-angled triangles.	Calculating midpoint of a line segment on a Cartesian plane.	Calculating gradient of a line segment on a Cartesian plane.	Determining distance between two points on the Cartesian plane.

Teaching and learning intentions and assessment	Suggested learning experiences
to provide instant feedback.	(Lesson 11)

Applying Pythagoras' Theorem in a familiar context and two step triangles.	Calculating midpoint of a line segment from coordinates and determining the end point when given the midpoint and other end point.	Calculating gradient of a line segment from coordinates and applications in contextual problems.	Determining distance between two points from coordinates and applications in contextual problems.

Students progress from the station with questions at the expected Standard to the next station on the same topic. If students are having difficulty with the questions at the expected Standard, they take a photo using their device or access this digitally, continuing to work on it during the time allocated at the second station.

Extension opportunities

- If students are demonstrating that they are well above the expected Standard during this sequence of lessons, create an additional set of stations which match the activities in the table below. These students can choose to skip the questions at the expected Standard if they are confident with the skills. The teacher determines if these students are ready to progress further.

Applying Pythagoras' Theorem to unfamiliar or changing contexts with problems which require more than one calculation.	Solving multi-step problems involving the midpoint of a line segment. Mixed problems involving midpoint, distance and gradient.	Determining the coordinates of an endpoint given the gradient and distance. Mixed problems involving midpoint, distance and gradient.	Determining the differences in distances of a moving object. Mixed problems involving midpoint, distance and gradient.

Lesson 12: Summative assessment

Western Australian curriculum content

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Teaching and learning intentions and assessment	Suggested learning experiences (Lesson 12)
Learning intentions • Provide feedback to teacher and students on their development and understanding during the course of this unit.	**Sequence** 1. Students are to complete the Summative assessment task – Pythagoras' TV-rem. (Appendix C) 2. Students who finish early write a reflection on how they have performed during this unit, what they think they did well, what would have helped them develop a deeper understanding and what they found interesting about the topics covered.

APPENDIX A:
TEACHING AND LEARNING – RESOURCES

Appendix A.1 | Resources

Lesson	Link/information
1	Math-Aids.Com. (2021). *Math worksheets | Dynamically created math worksheets.* Retrieved May, 2021, from https://www.math-aids.com/ Graphing Worksheets for Practice (Math-Aids.com)
	Street. (n.d.). *Mr. Street's geeky graphs.* Google Sites. Retrieved May, 2021, from https://sites.google.com/site/mrstreetsgeekygraphs/home
	WorksheetWorks.com. (2021). *Coordinate picture graphing*. Retrieved May, 2021, from https://www.worksheetworks.com/math/geometry/graphing/coordinate-pictures.html
2	Corbett, J. (2021). *Corbettmaths | Videos, worksheets, 5-a-day and much more.* Retrieved May, 2021, from https://corbettmaths.com/ Videos and Worksheets | Coordinates: midpoint of a line (Corbett Maths)
	EasyTeacherWorksheets.com. (2021). *Easy teacher worksheets – ready to print teacher worksheets.* Retrieved May, 2021, from https://www.easyteacherworksheets.com/ Midpoint of a Line Segment Worksheets (Easy Teacher Worksheets)
	EdPlace (2021). *The Smartest Revision App - Improve Grades - Build Confidence.* Retrieved May, 2021, from https://www.edplace.com/ Finding the Coordinates of the Midpoint of a Line Segment (edplace)
	Kuta Software LLC. (2021). *Create Custom Pre-Algebra, Algebra 1, Geometry, Algebra 2, Precalculus, and Calculus worksheets.* Retrieved May, 2021, from https://www.kutasoftware.com/ Free Pre-Algebra Worksheets | Plane Figures | The midpoint formula (Kuta Software)
	Mathworksheets4kids. (2021). *Worksheets for kids | Free printables for K-12*. Retrieved May, 2021, from https://www.mathworksheets4kids.com/ Midpoint Formula Worksheets (MATH Worksheets 4 Kids)
	Tutoringhour. (2021). *Teaching resources | Worksheets for kids*. Retrieved May, 2021, from https://www.tutoringhour.com/ Midpoint of a Line Segment Worksheets (Tutoring Hour)
4	Vennebush, P. (2021). *Geoboard by the math learning center.* Retrieved May, 2021, from https://apps.mathlearningcenter.org/geoboard/
5	IXL Learning. (2021). *IXL | Maths and English practice.* Retrieved May, 2021, from https://au.ixl.com/

Lesson	Link/information
	▪ Pythagoras' Theorem: find the length of the hypotenuse (IXL Learning)
	Khan, S. (2021). *Free online courses, lessons and practice.* Khan Academy. Retrieved May, 2021, from https://www.khanacademy.org/ ▪ Pythagorean Theorem 1 (Khan Academy)
	Pierce, R. (2017). *Math is Fun.* Retrieved May, 2021, from https://www.mathsisfun.com/ ▪ Pythagoras' Theorem (MathsIsFun)
	Rodriguez-Clark, R. (2019). *Interactive Maths – the interactive way to teach Mathematics.* Retrieved May, 2021, from https://www.interactive-maths.com/ Pythagoras Proof [GGB] (Interactive Maths) Pythagoras Theorem [GGB] (Interactive Maths)
	The Math Salamanders. (n.d.). *Math worksheets education from The Math Salamanders.* Retrieved May, 2021, from https://www.math-salamanders.com/ ▪ Pythagoras Theorem questions (Math-Salamanders)
6	Andreajn at Facts.net. (2020, December 22). *Pythagoras facts.* Retrieved May, 2021, from https://facts.net/history/people/pythagoras-facts/ ▪ 50 surprising Pythagoras facts you never knew (facts.com)
	Australian Academy of Science. (2020). *reSolve | Promoting a spirit of enquiry.* Retrieved May, 2021, from https://resolve.edu.au/ ▪ Geometry: Lunch Lap (reSolve)
	Emma at SuperProf. (2019). *Superprof | Home tutoring & private tutoring*. Retrieved May, 2021, from https://www.superprof.co.uk/ ▪ Pythagorean theorem word problems (Superprof)
	Hall, J. (2021). *StudyMaths.co.uk - GCSE maths revision*. Retrieved May, 2021, from https://studymaths.co.uk/ ▪ Worksheet – Worded Pythagoras' Theorem questions (StudyMaths.co.uk)
	Hart, V. (2012, June 13). *Mystery man Pythagoras meets his match* [Video]. ABC Education. Retrieved May, 2021, from https://education.abc.net.au/home#!/media/1003922/ ▪ Mystery man Pythagoras meets his match (ABC Education)
	IXL Learning. (2021). *IXL | Maths and English practice.* Retrieved May, 2021, from https://au.ixl.com/ Pythagoras' Theorem word problems (IXL Learning)

Lesson	Link/information
	Khan, S. (2021). *Free online courses, lessons and practice.* Khan Academy. Retrieved May, 2021, from https://www.khanacademy.org/ ▪ Pythagorean Theorem word problems basic (Khan Academy)
	TemplateLab. (2021). *TemplateLab | Best legal and business templates.* Retrieved May, 2021, from https://templatelab.com/ ▪ 48 Pythagorean Theorem worksheet with answers (TemplateLAB)
	Tranter, J. (n.d.). *Transum*. Retrieved May, 2021, from https://www.transum.org/ ▪ Pythagoras' Theorem Exercise (Transum)
9	Brasz, F. (n.d.). *Voronoi diagram area game.* Retrieved May, 2021, from https://cfbrasz.github.io/VoronoiColoring.html ▪ Voronoi diagram area game (github)
	Khan, S. (2021). *Free online courses, lessons and practice.* Khan Academy. Retrieved May, 2021, from https://www.khanacademy.org/ Voronoi Partition [Video] (Khan Academy)
10	FEI. (2013, December 18). *Charlotte Dujadin's world record breaking freestyle - Reem Acra FEI World Cup™ Dressage 2013/14* [Video]. YouTube. Retrieved May, 2021, from https://www.youtube.com/watch?v=tcIzsC7_Has
Formative	BetterExplained. (n.d.). *Surprising uses of the Pythagorean Theorem*. Retrieved May, 2021, from https://betterexplained.com/articles/surprising-uses-of-the-pythagorean-theorem/
	Maths Advice On Your Device. (2020, January 29). *Pythagoras' Theorem using other shapes (Ep.2)* [Video]. YouTube. Retrieved May, 2021, from https://www.youtube.com/watch?v=6rCdvPI40R4

Appendix A.2 | Cartesian coordinates figures

Lesson 1

Instructions for teacher

Use this game for students to practise their placement and reading of Cartesian coordinates. Students draw a series of figures of different shapes and sizes onto their game board. Each player takes turns in guessing a set of coordinates to try and completely locate each figure. This game is based on the classic board games with a similar premise.

Students work in pairs with one game sheet each with a divider up between them, such as a file or laptop screen. The game can be extended to involve larger groups as indicated in the optional rules at the end of the task sheet.

This game can be modified for students who are not at the expected Standard by only including the first quadrant.

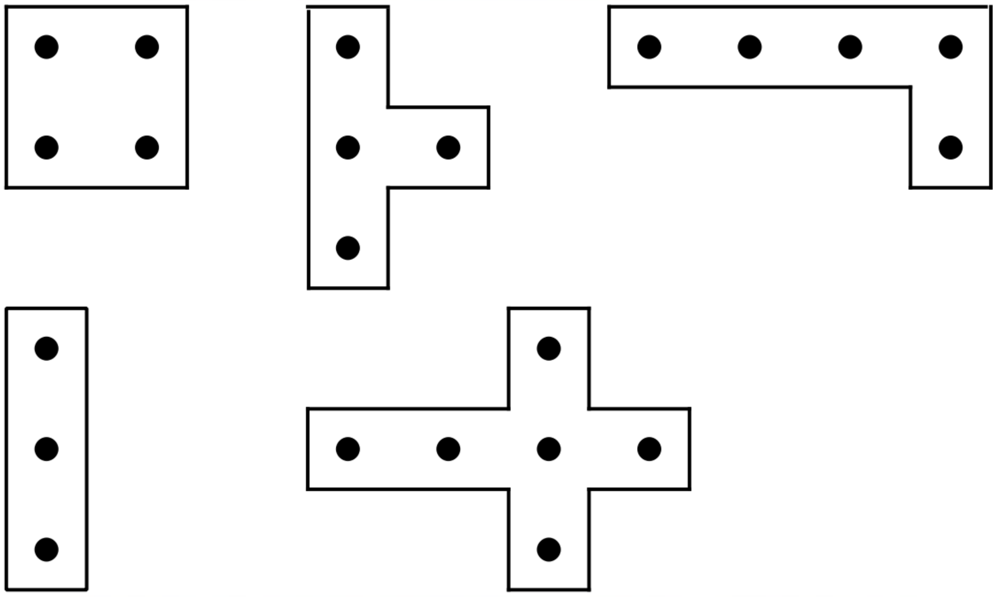
Learning task: Cartesian coordinate figures

Playing against your partner, you will place the figures shown below onto the Cartesian plane labelled My Board. The dots in the middle of the shapes represent the coordinates. Place the dots only on integer coordinates.

Each player takes turns in being the guesser. The guesser states a coordinate and the other player responds if it is a hit or a miss. The players both mark where the guess was on the appropriate board. The other player becomes the guesser and they repeat the process until one player has guessed all of the coordinates of the other player.

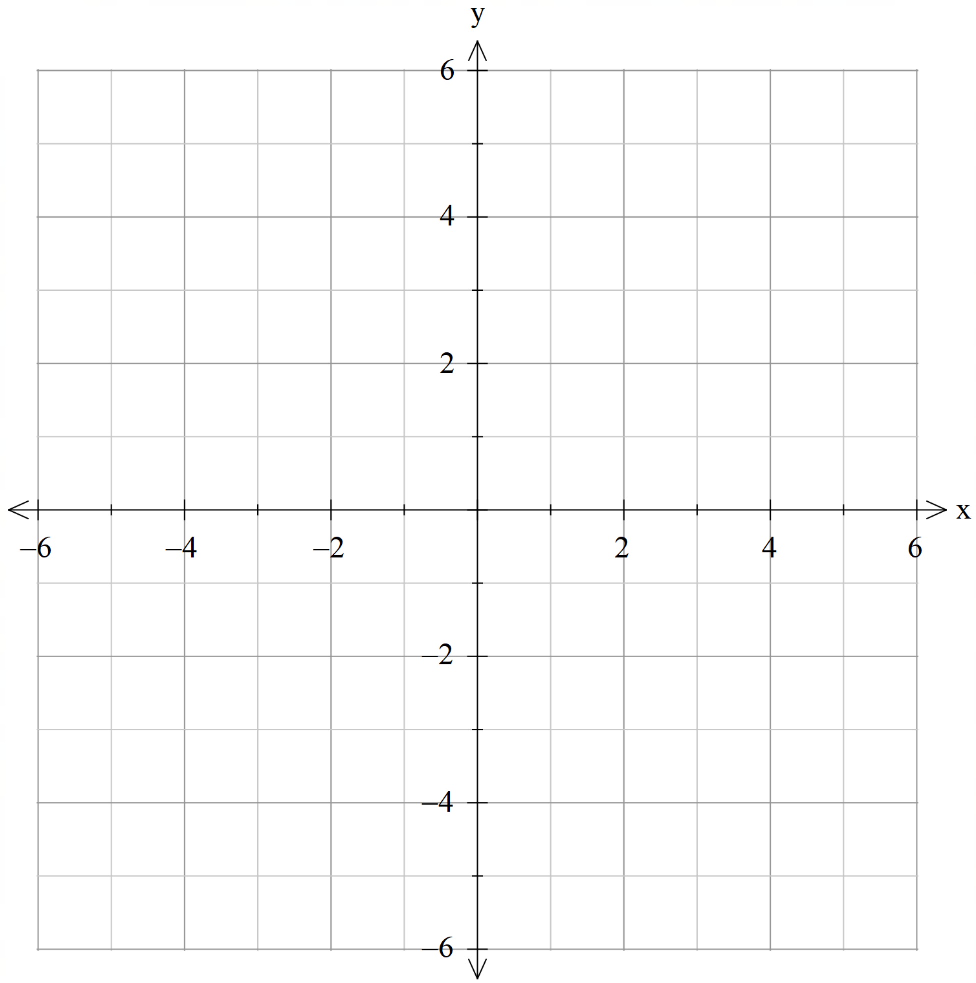
After a shape has been completed tell the other person they have guessed a whole shape.

Figures to use

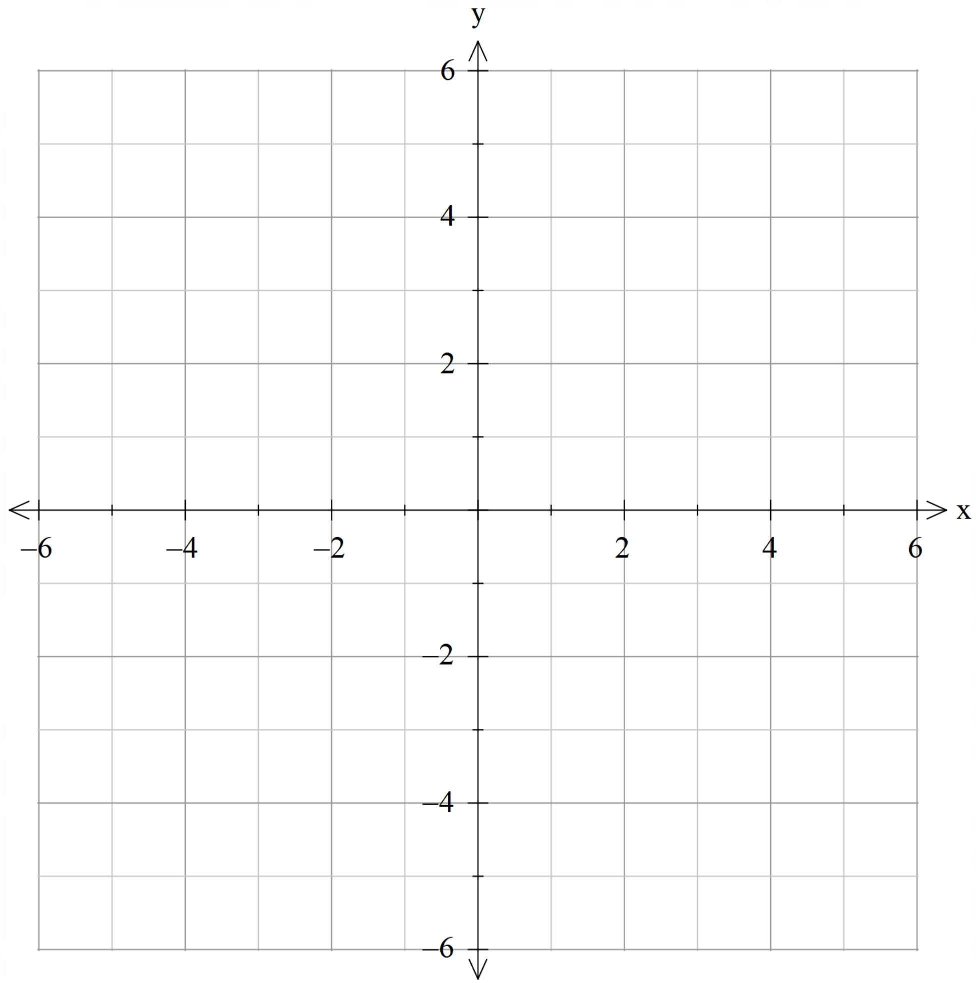


Optional rules

- You can both add an extra shape with four dots, but it must be connected using horizontal and vertical lines.
- You do not need to say when a whole shape has been covered.
- You can both add one specific coordinate which acts as a mirror. It acts as a guess on the other person's board, including the coordinates in each compass direction making a + shape.
- Add a third or fourth player to the group. Each person guessing states a coordinate which applies to all of the other people playing at once.



My Board



My Guesses

Appendix A.3 | Number lines

Lesson 2

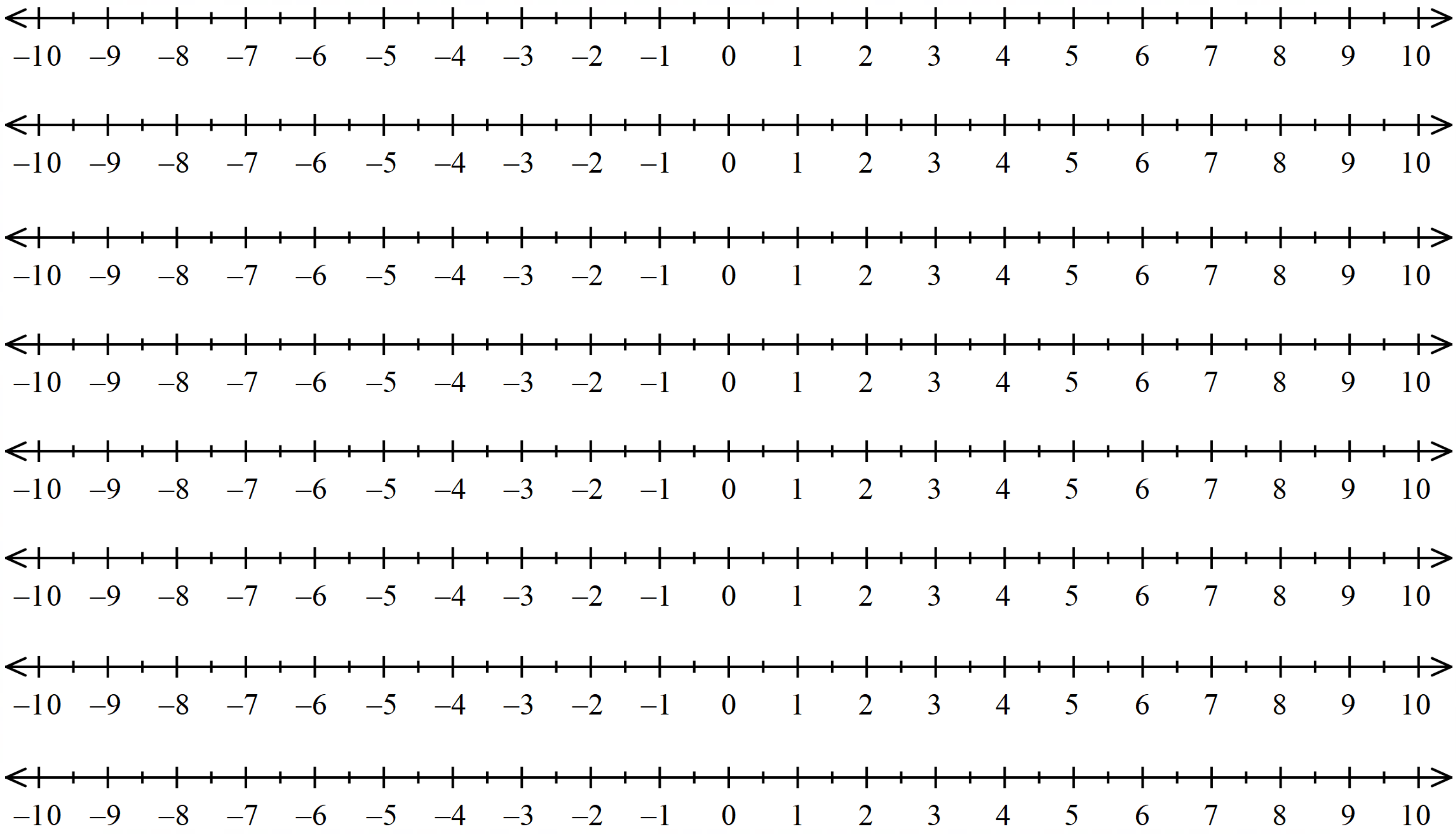
Instructions for teacher

These number lines are to be used in modelling the process of finding the middle of two integers. Students locate each number on the number line and then fold the number line in half to identify the exact middle of these two points.

After this activity, use this resource to support and scaffold students who are not at the expected Standard. It will help them to quickly find the middle value of two integers and should provide students with opportunities to develop skills around finding the mean of two numbers quickly.

This skill can be extended to a Cartesian plane when students begin to operate with the midpoint of Cartesian coordinates. Folding the horizontal and vertical distances will result in the midpoint as folding the coordinate from end to end would.

Learning resource – Number lines



Appendix A.4 | Cartesian plane and coordinates – midpoint

Lesson 2

Instructions for teacher

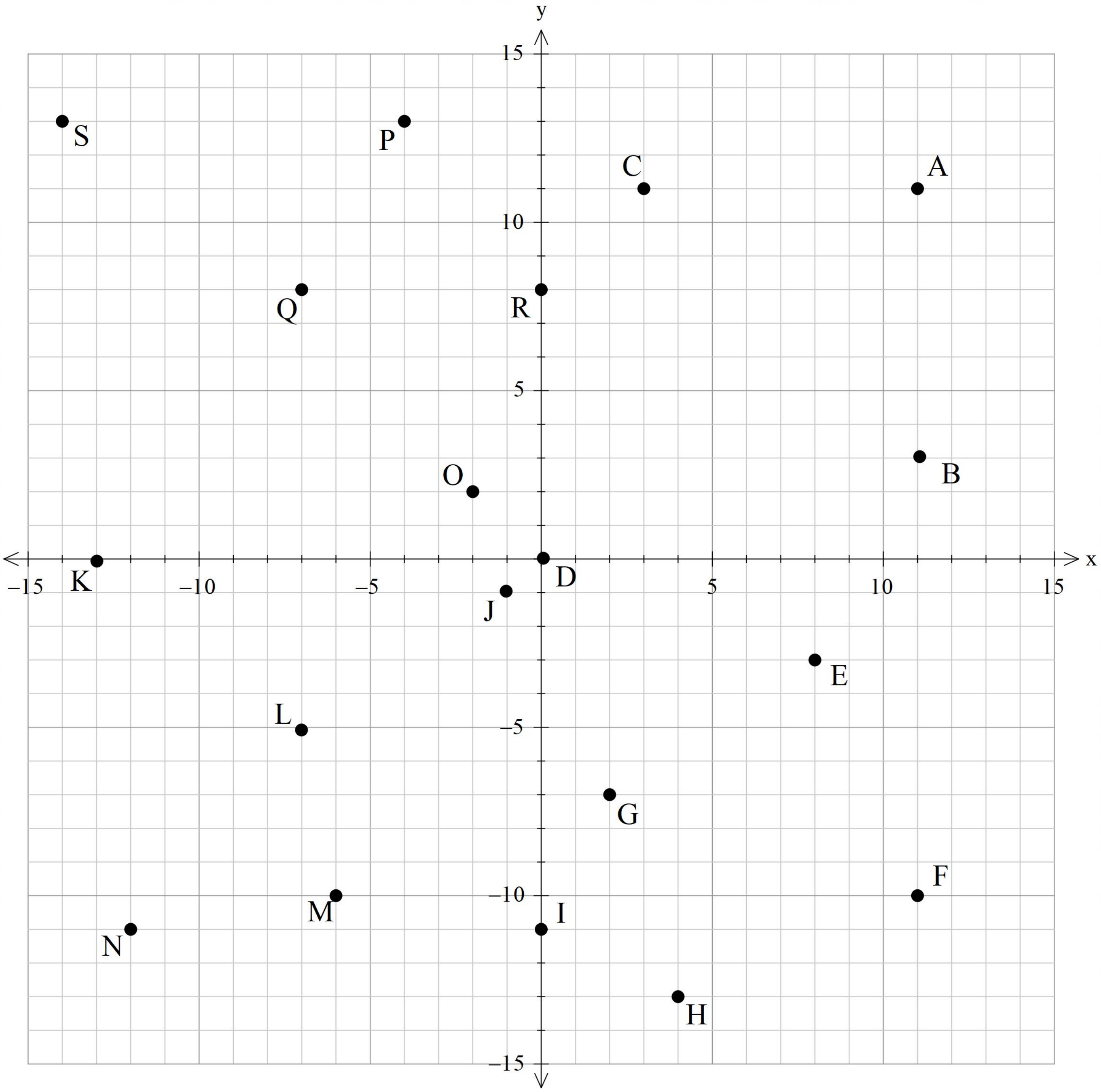
This activity provides students with an opportunity to explore the midpoint of coordinates in all four quadrants of the Cartesian plane. This can be open for the students to explore or guided by the teacher as required in the classroom context. Encourage students to challenge themselves by working in all four quadrants, working with numbers with a mix of odd and even coordinates and trying to figure out the pattern as quickly as possible.

Students may work together in groups to collect a larger amount of data quickly. It could be posed to the class to try and determine the midpoints of every pair of coordinates. This could result in a discussion about the best approach to try and get all of this information in an organised way.

This activity can be paired with the ideas presented in Appendix A.3 to get students folding their page to help identify the coordinates of the midpoint between two coordinates.

Learning task: Cartesian plane and coordinates – midpoint

Choose at least 10 different pairs of coordinates and determine the midpoint of these coordinates. You can choose any points you are confident working with. Write the name and coordinates of each point you are using in the table on the reverse of this page. Once you are confident finding the midpoint by inspecting the graph, look at the pairs and the midpoint and try to see if there are any patterns in the numbers.



First point	*x*-coordinate	*y*-coordinate	Second point	*x*-coordinate	*y*-coordinate	Midpoint	*x*-coordinate	*y*-coordinate
A (11, 11)	11	11	B (11, 3)	11	3	(11, 7)	11	7

How could you find the midpoint between any two coordinates?

Appendix A.5 | Centre of gravity

Lesson 3

Instructions for teacher

This activity looks at extending students' understanding of midpoint of a line and applying these rules to two-dimensional figures to determine the centre of gravity.

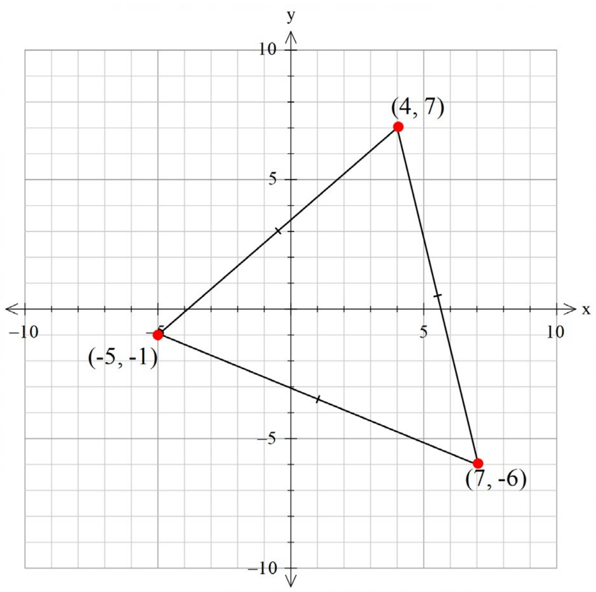
A range of shapes displayed on a Cartesian plane have been provided. Students cut them out and determine the point where they can balance them on their finger, marking this as the centre of gravity. Students then explore the shapes to determine a method to geometrically or algebraically calculate the centre of gravity.

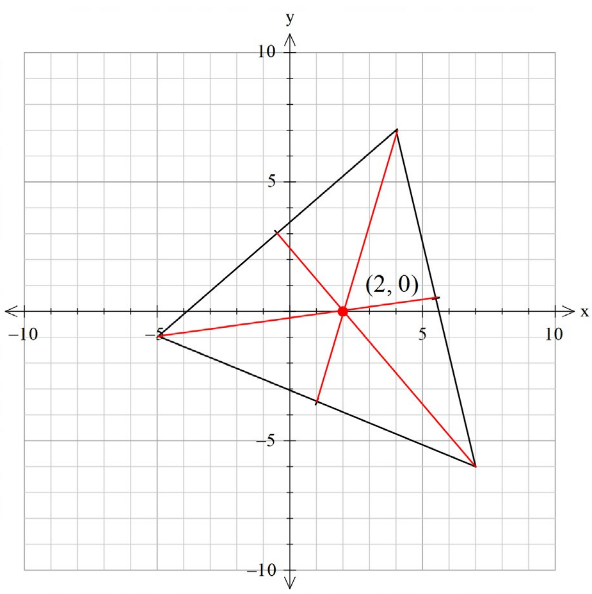
Students work with triangles first, then look at simple, then complex, quadrilaterals. If students are able to comfortably determine the centre of gravity of quadrilaterals, they can explore polygons with more sides.

Behind the mathematics

The centre of gravity of a triangle can be calculated algebraically by determining the mean of the *x*-coordinates of each vertex and the mean of the *y*-coordinates of each vertex. The combination of these two values as a pair of coordinates gives the centre of gravity. Students can use this method to check their answer, if they discover it.

The geometric approach to determining the centre of gravity utilises the midpoint of each line segment making up an edge of the triangle. These midpoints are used to draw a line segment to the opposing vertex. Where the three lines intersect is the centre of gravity. This process is shown below.

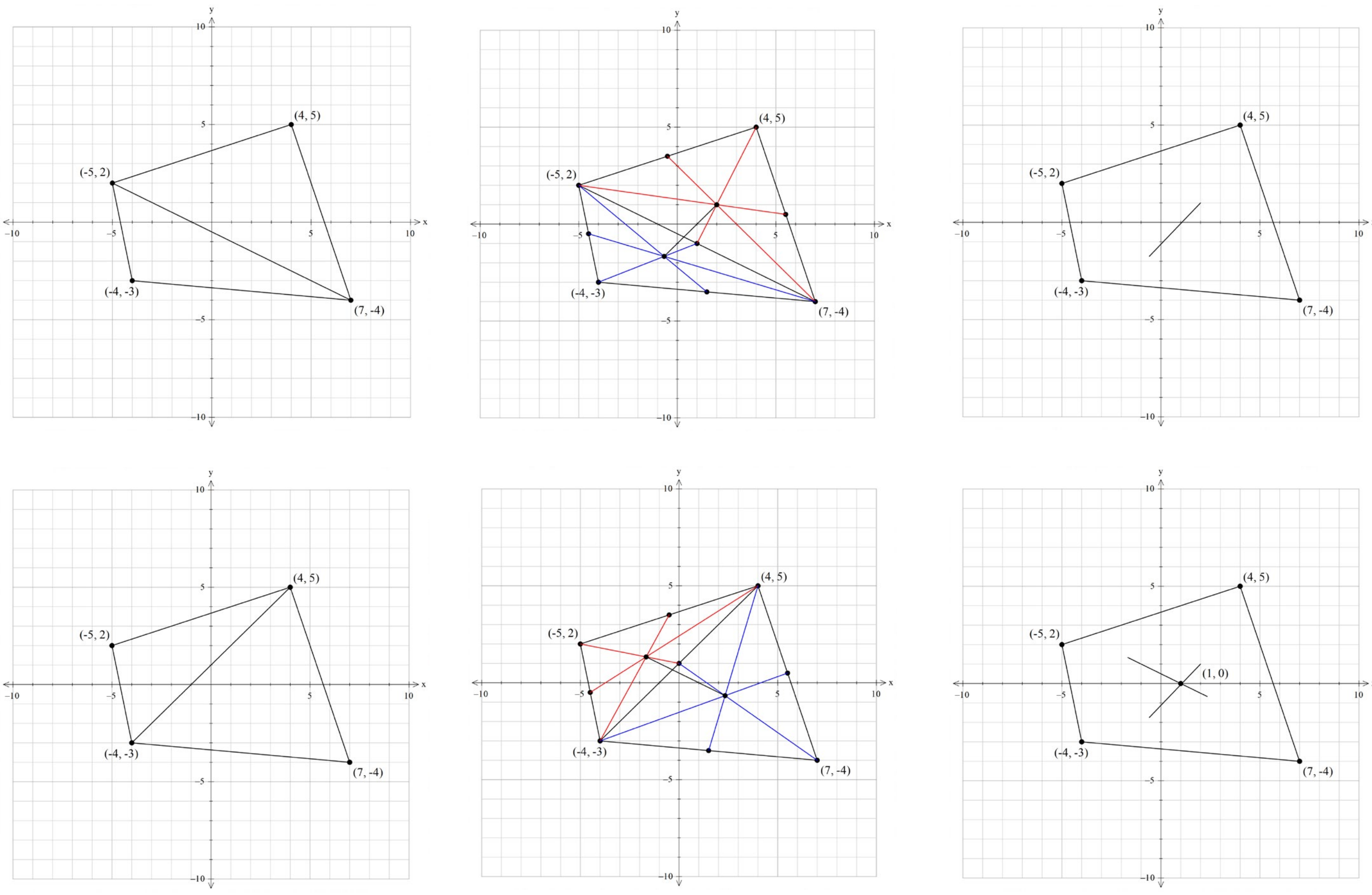




Geometrically, the centre of gravity is at the point (2, 0). This can be checked algebraically. The mean of the *x*-coordinates is $\frac{-5+4+7}{3} = \frac{6}{2} = 2$ and the *y*-coordinates is $\frac{-1+7+(-6)}{3} = 0$. Writing these as coordinates gives $(2, 0)$ which confirms the solution.

To determine the centre of gravity of a quadrilateral geometrically, the shape has to be broken into two triangles. This is done by connecting one vertex to the diagonally opposite vertex. Connecting the centre of gravity of these triangles gives the line where the centre of gravity occurs. To determine it exactly, the quadrilateral must be split into triangles using the opposite vertices, with their centres of gravity connected. The point of intersection of these two lines is the centre of gravity of the quadrilateral.

This process is demonstrated below.



Extension: if students want to extend themselves to determine the centre of gravity of pentagons and beyond, they will need to use these same ideas to determine the centres of gravity of smaller shapes, and then find where these centres of gravity intersect. A pentagon will need to be split into a triangle and a quadrilateral using one pair of opposing vertices with their centres of gravity connected, and then repeated for a different triangle and quadrilateral pairing.

It is important for students to calculate the midpoints of the line segments they are working with at all stages of the process.

Learning task – Centre of gravity

In your groups of four, you will be investigating how to find the centre of gravity of a range of figures. You will first be trying to balance the figures provided on your finger by a single point, then trying to use algebra or geometry to determine where that point is. During this activity, keep a focus on what you have learnt in the previous lessons, and try to apply these skills to this activity.

Part 1 – Balancing

You will be provided with a range of different figures drawn on Cartesian planes. Cut these out and try to determine the point where you can balance this figure on your finger tip. This location is known as the centre of gravity.

Write the coordinates of the vertices as well as the estimate of the coordinates of the centre of gravity of these shapes. What do you notice?

Part 2 – Triangles

In your groups, use your results to research and explore some more triangles. Try to determine a method for finding the centre of gravity, or centroid, of any triangle.

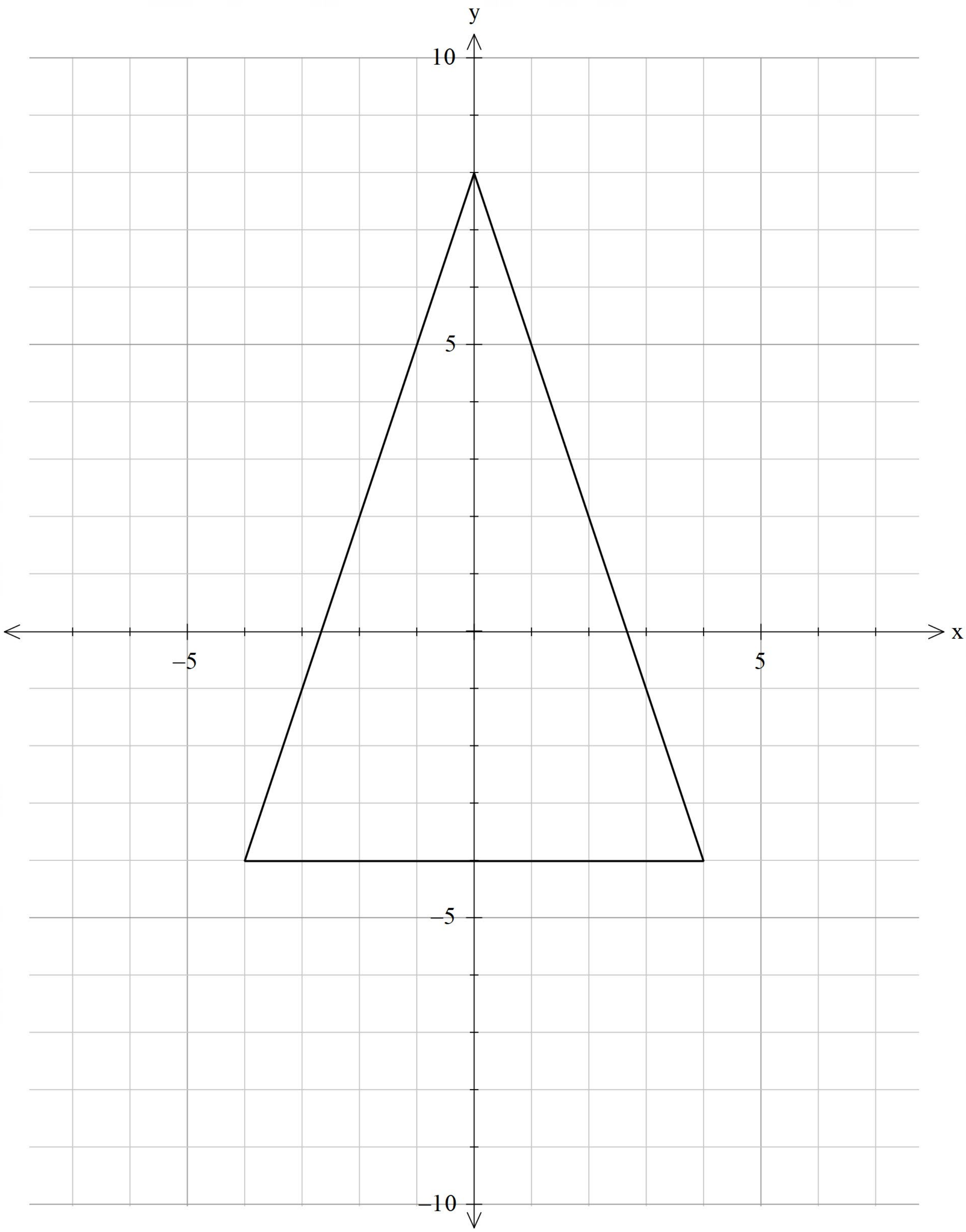
Hint: use the midpoints of each line segment or think about how the midpoint is calculated in general.

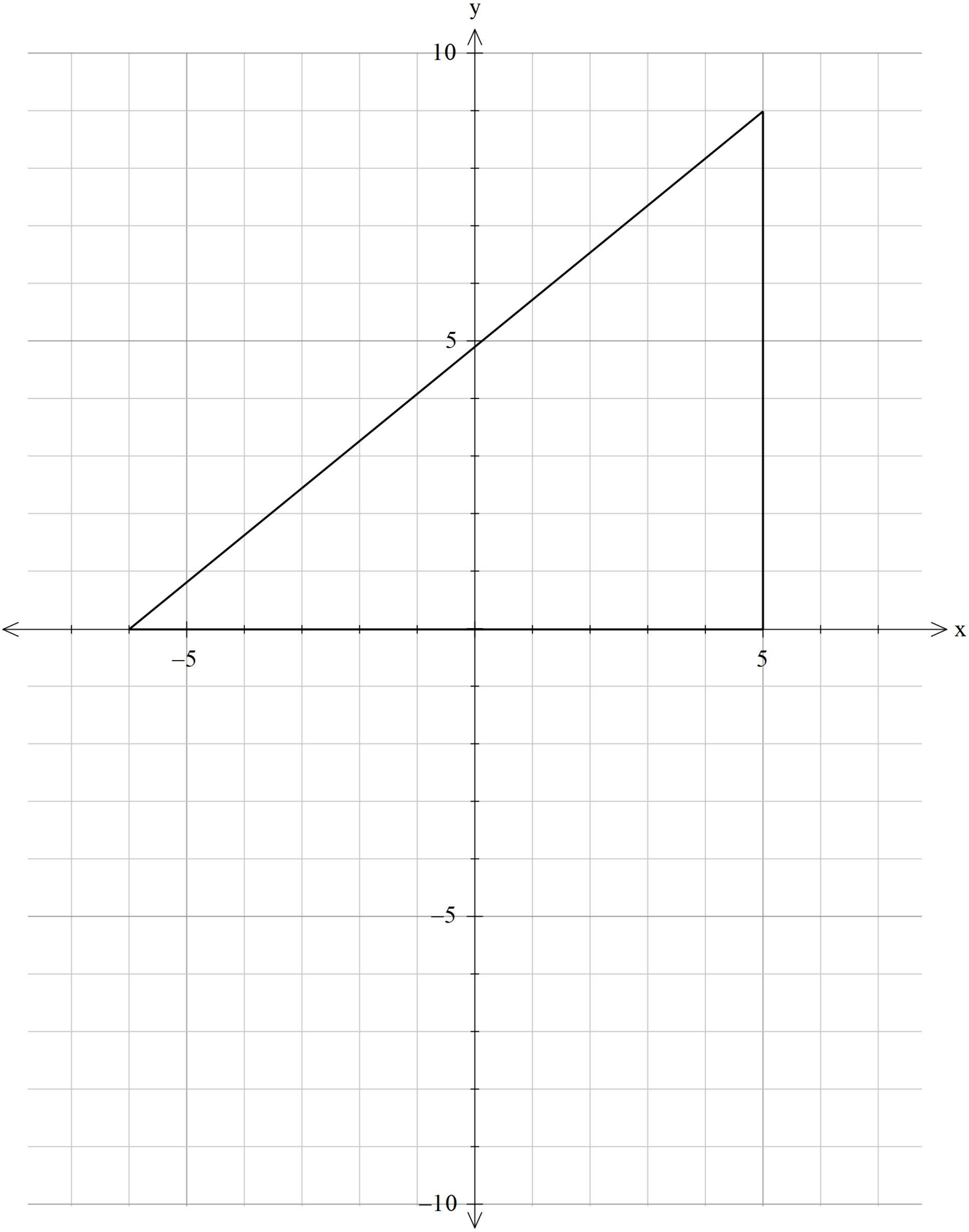
Part 3 – Quadrilaterals

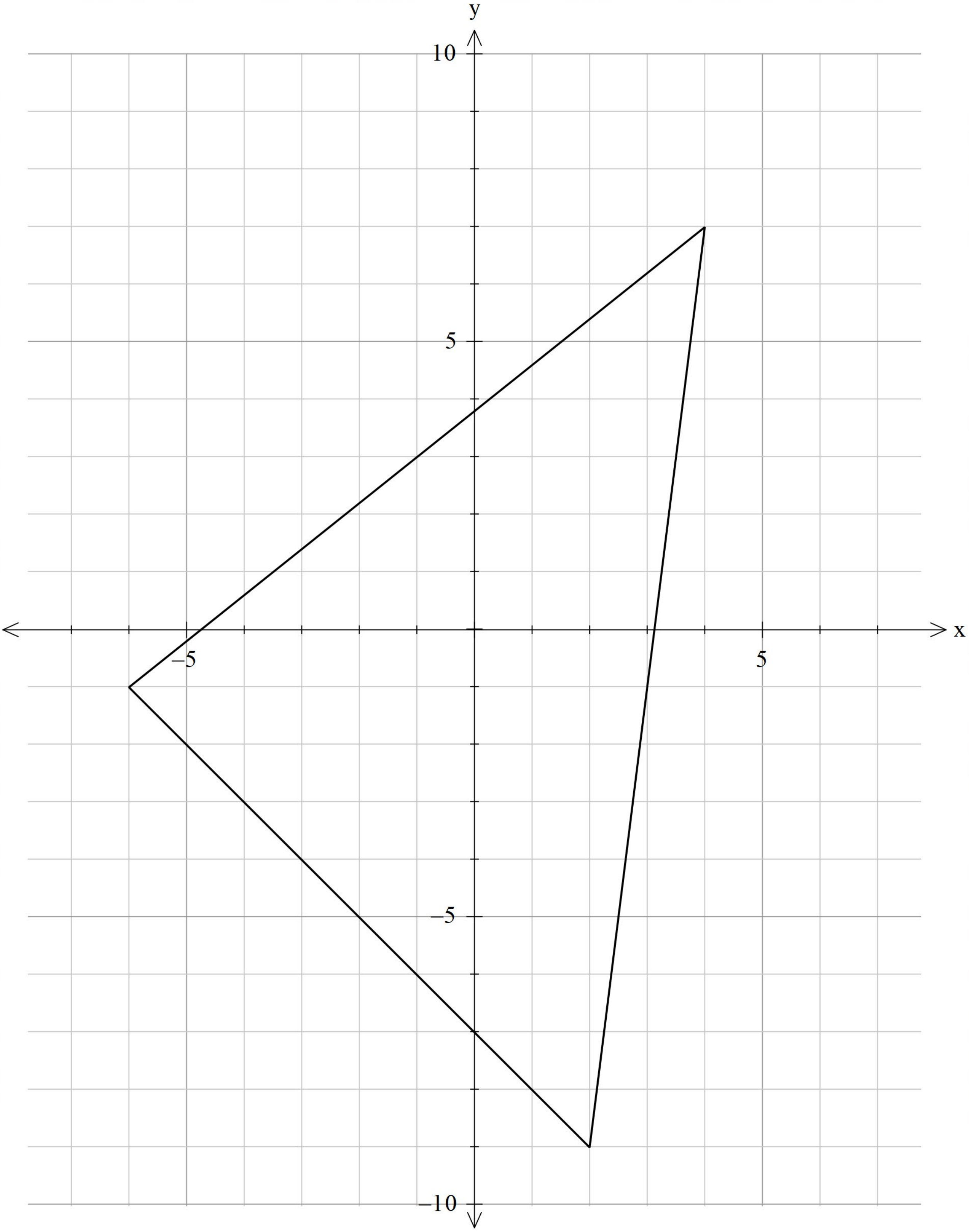
Once you have determined how to work out the centre of gravity of any triangle, investigate further by experimenting with some quadrilaterals. Determine the centre of gravity, using the midpoint of the line segments.

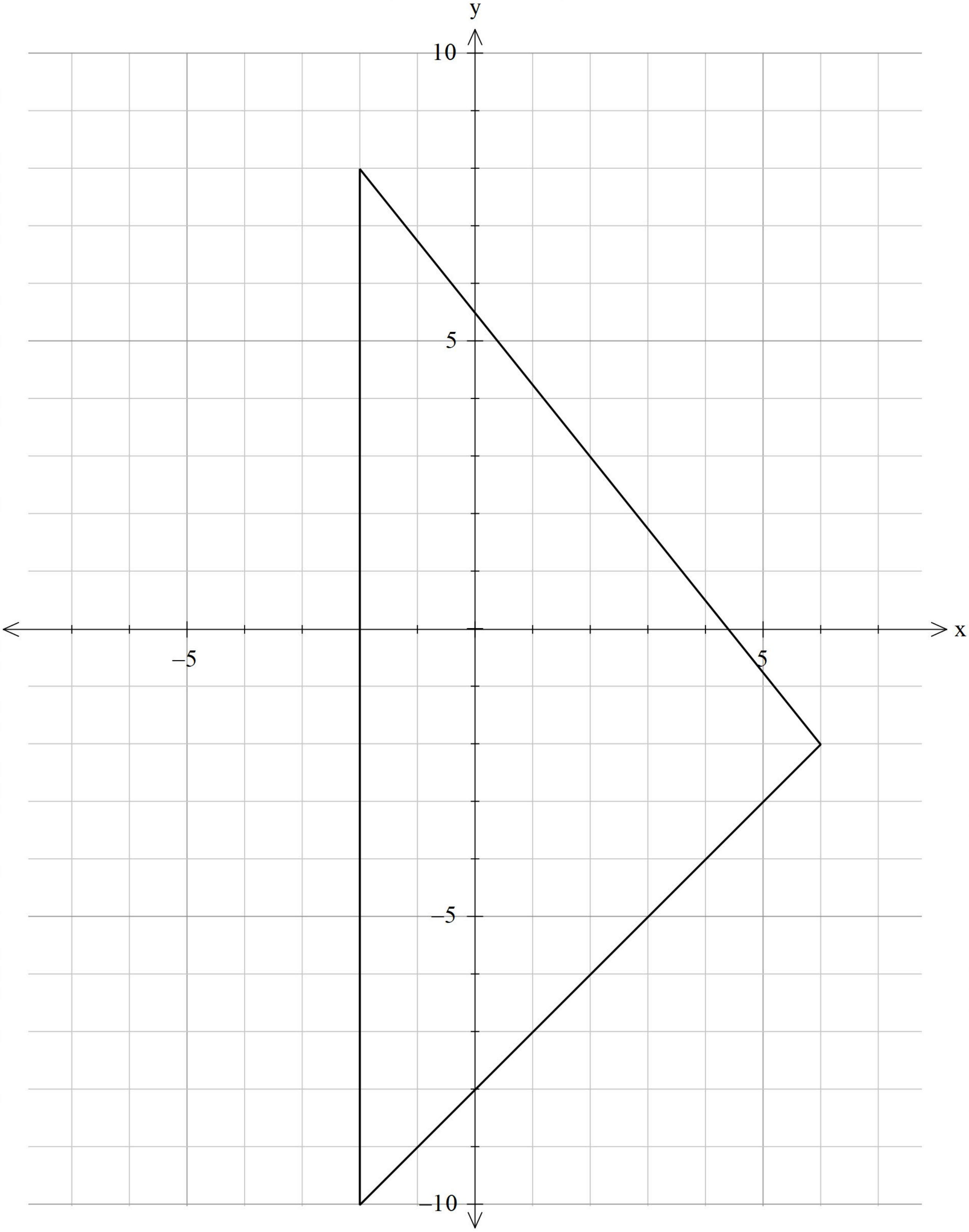
If you get stuck, look online, consult your teacher or use the hint below.

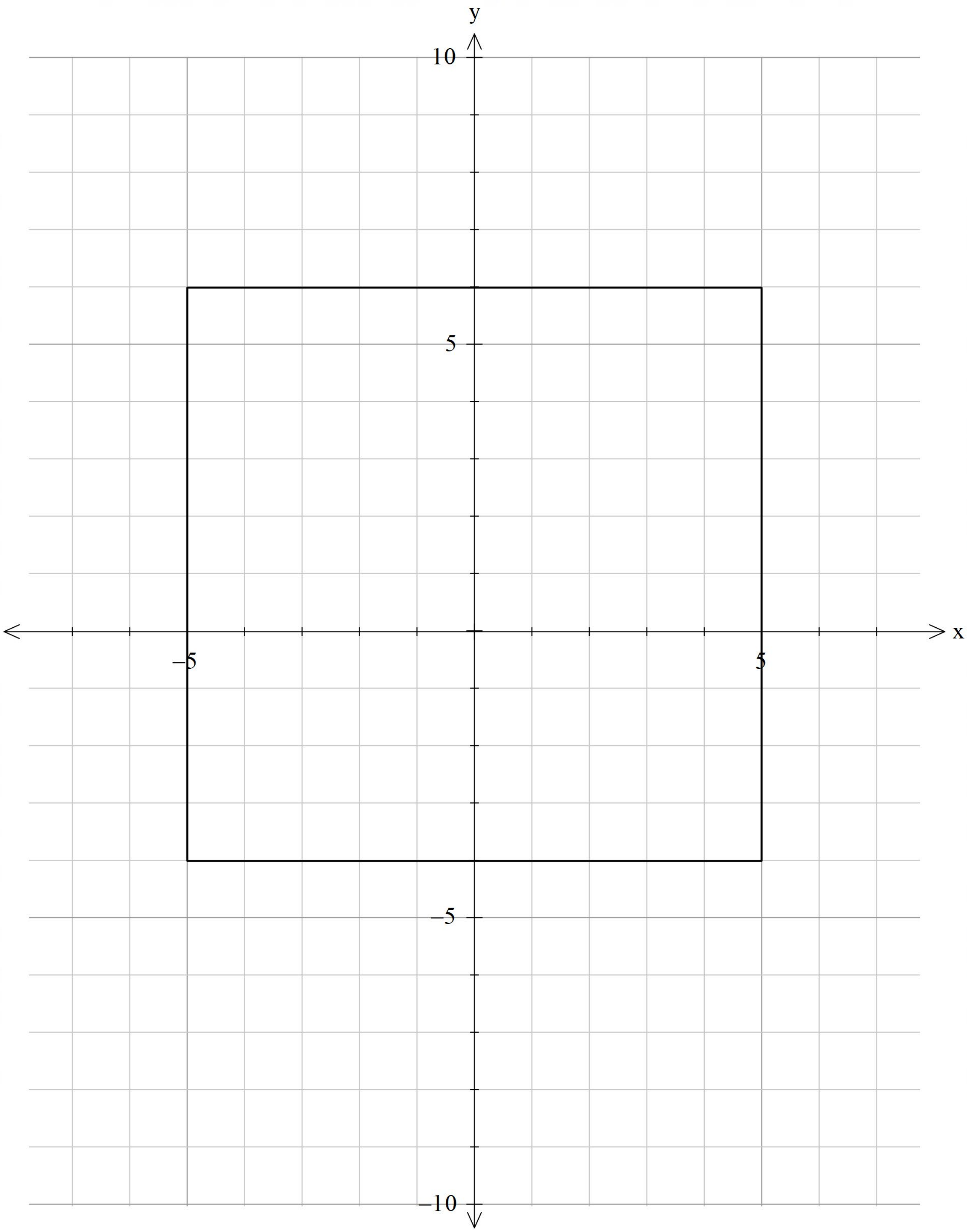
Hint: try breaking your shape into triangles.

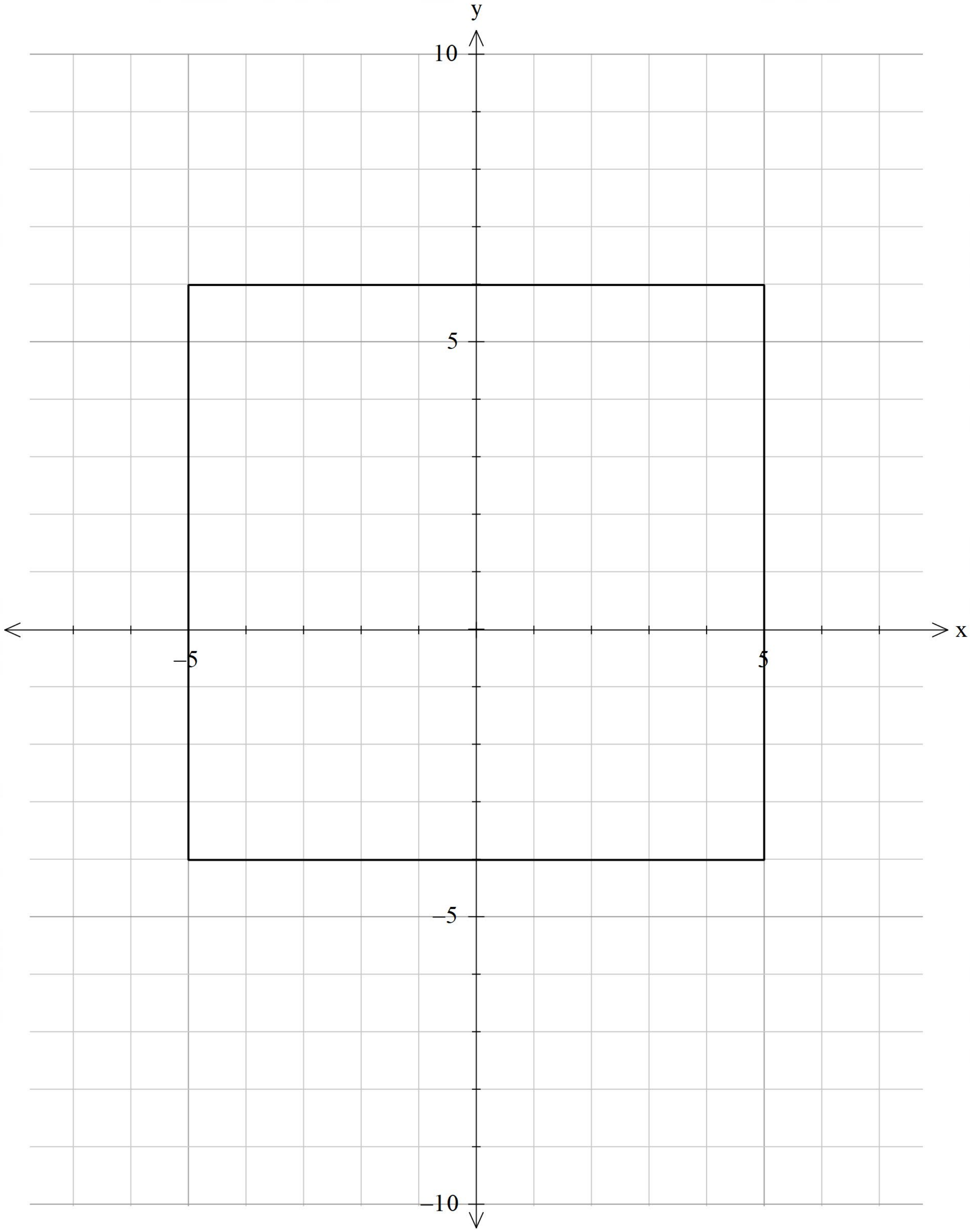

y
10
5
x
–5
5
–5
–10

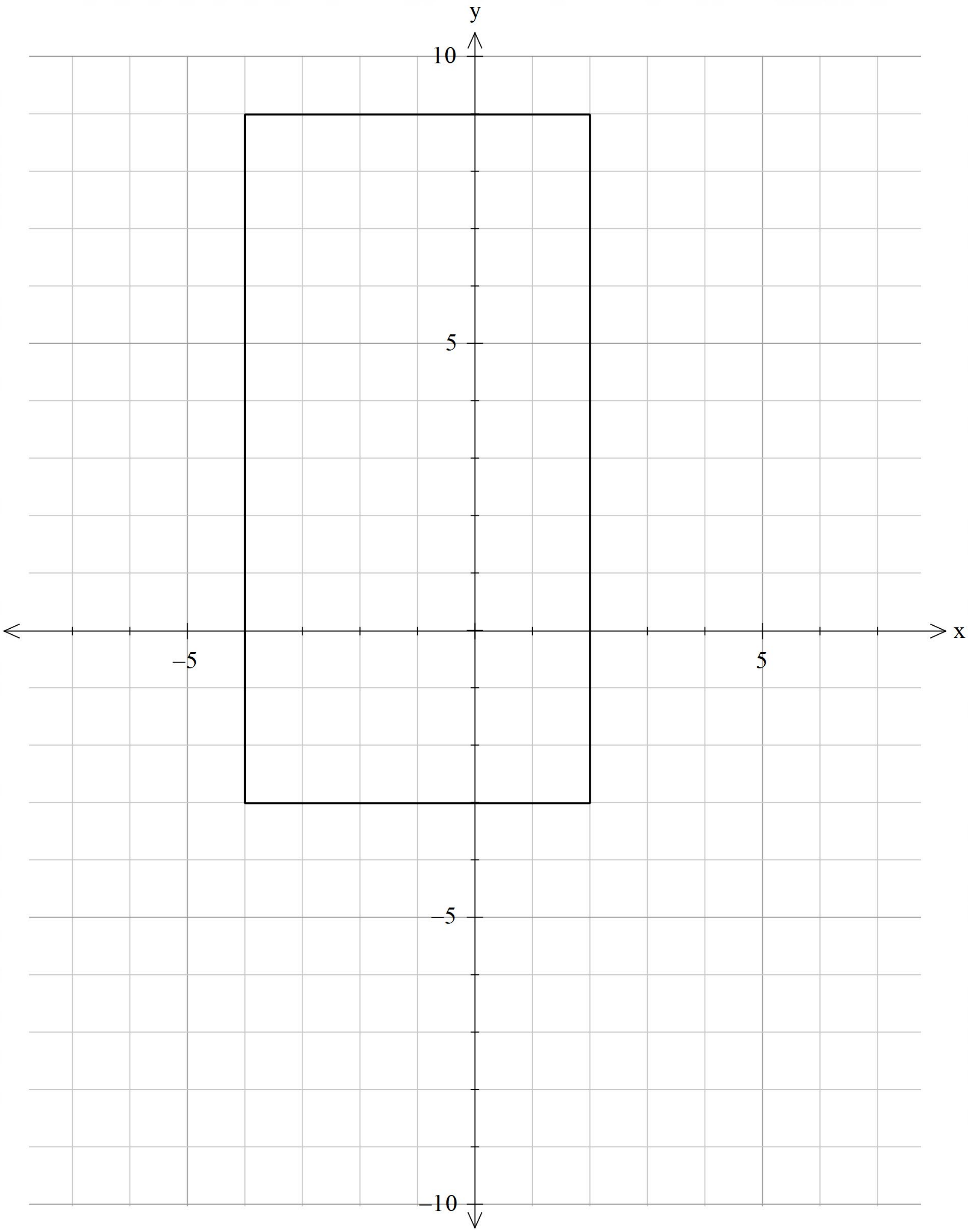

y
10
5
–5
–10
x
–5
5

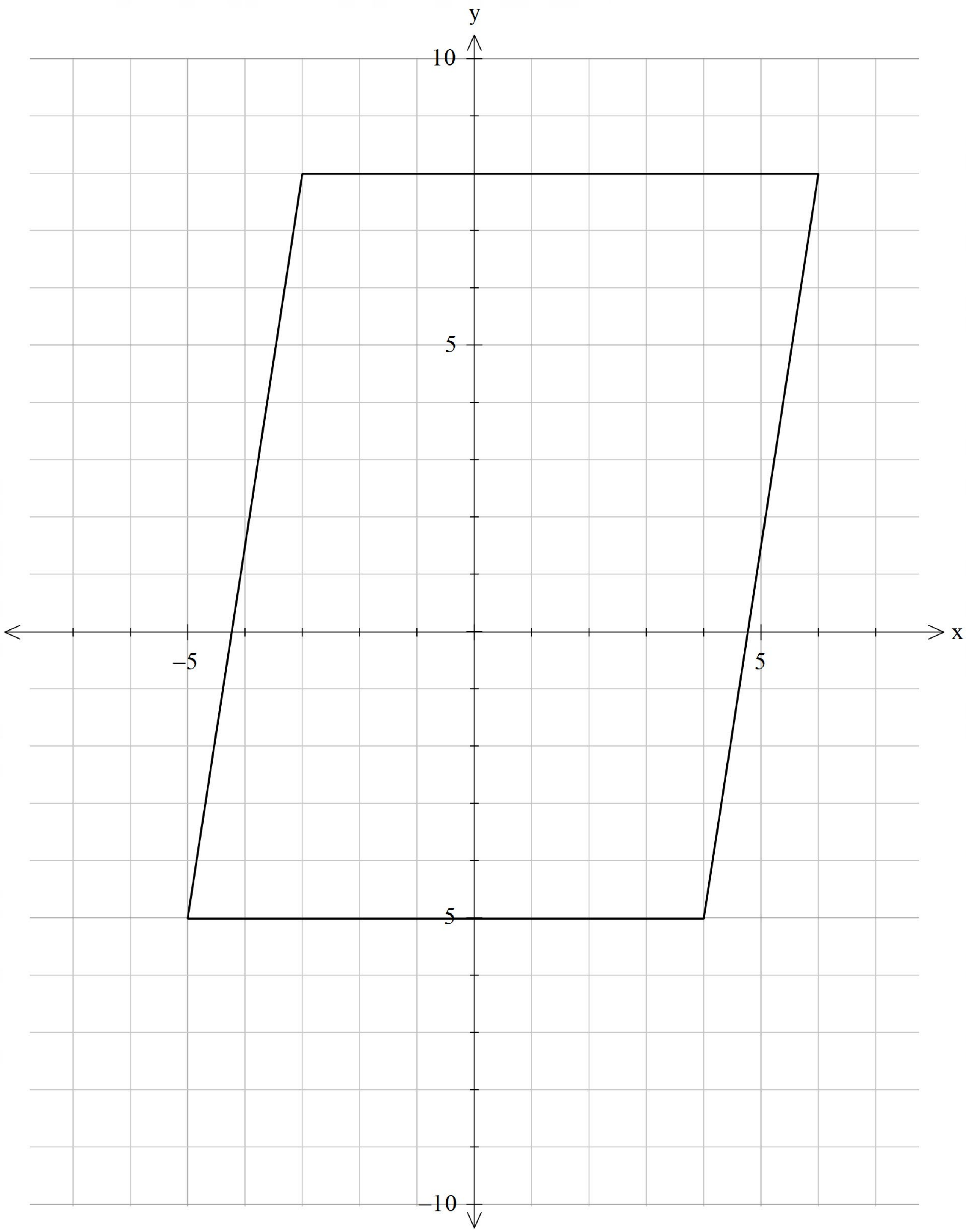

y
10
5
x
–5
5
–5
–10

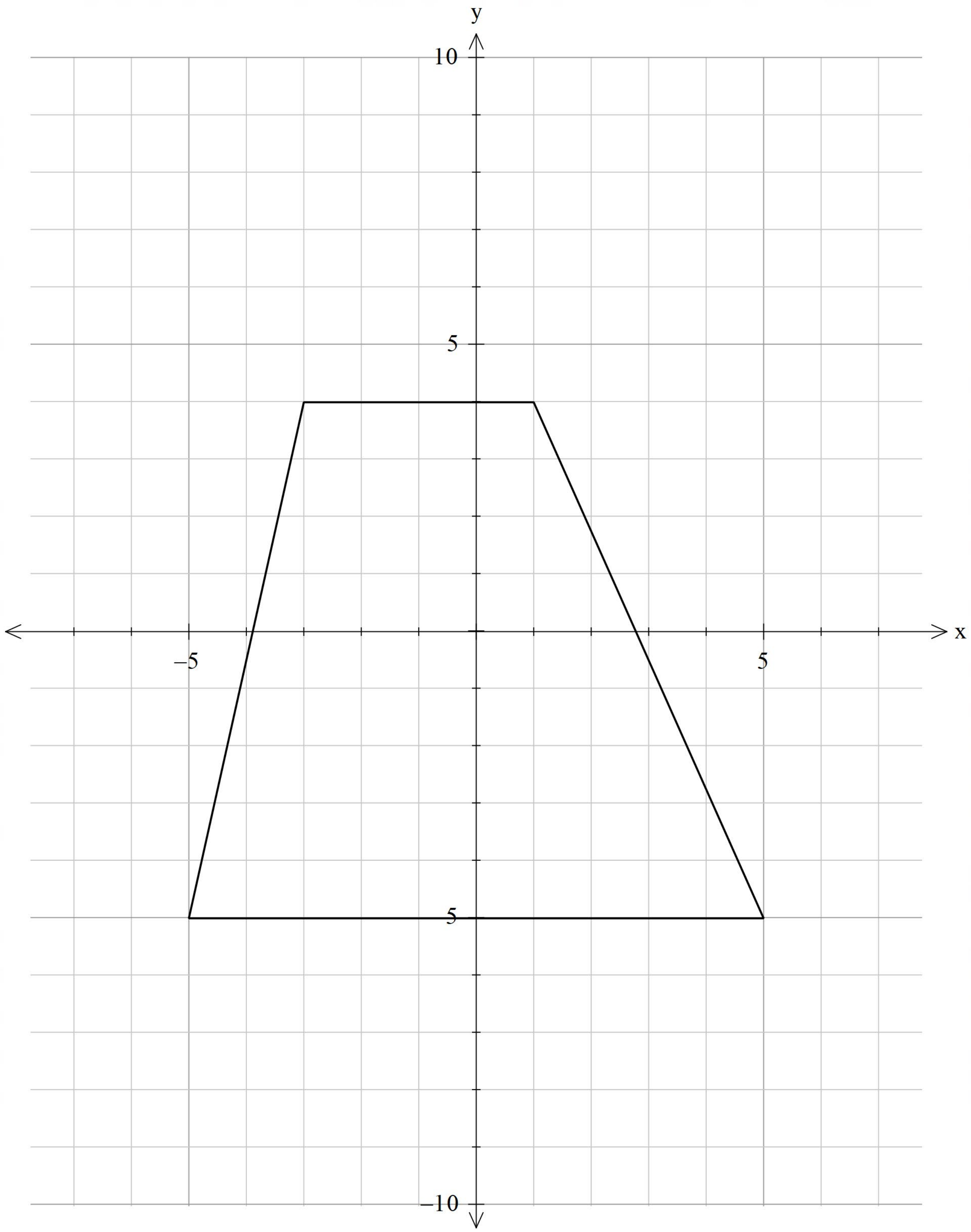

y
10
5
–5
–10
x
–5
5

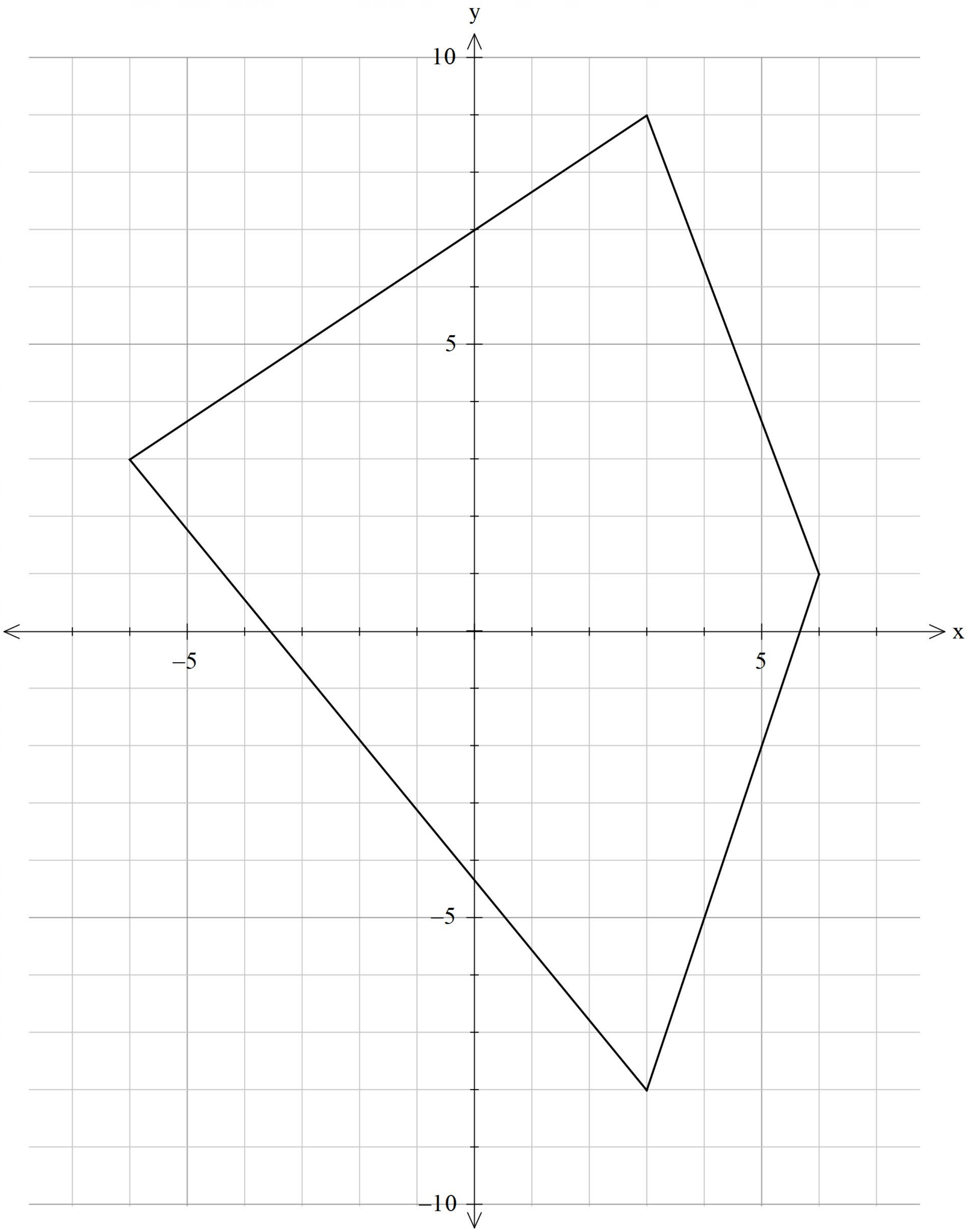

y
10
5
x
–5
5
–5
–10

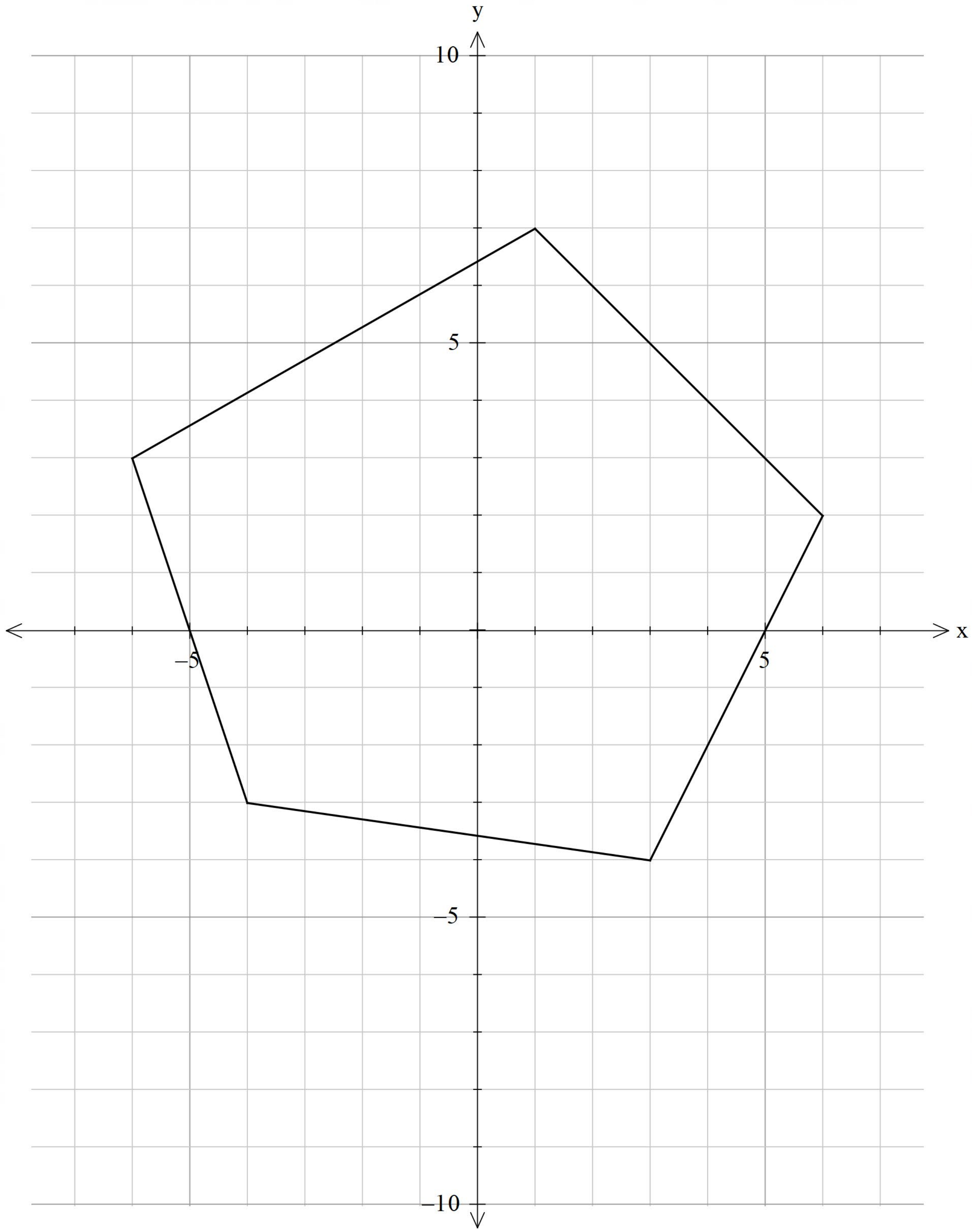

y
10
5
–5
5
x
–5
–10

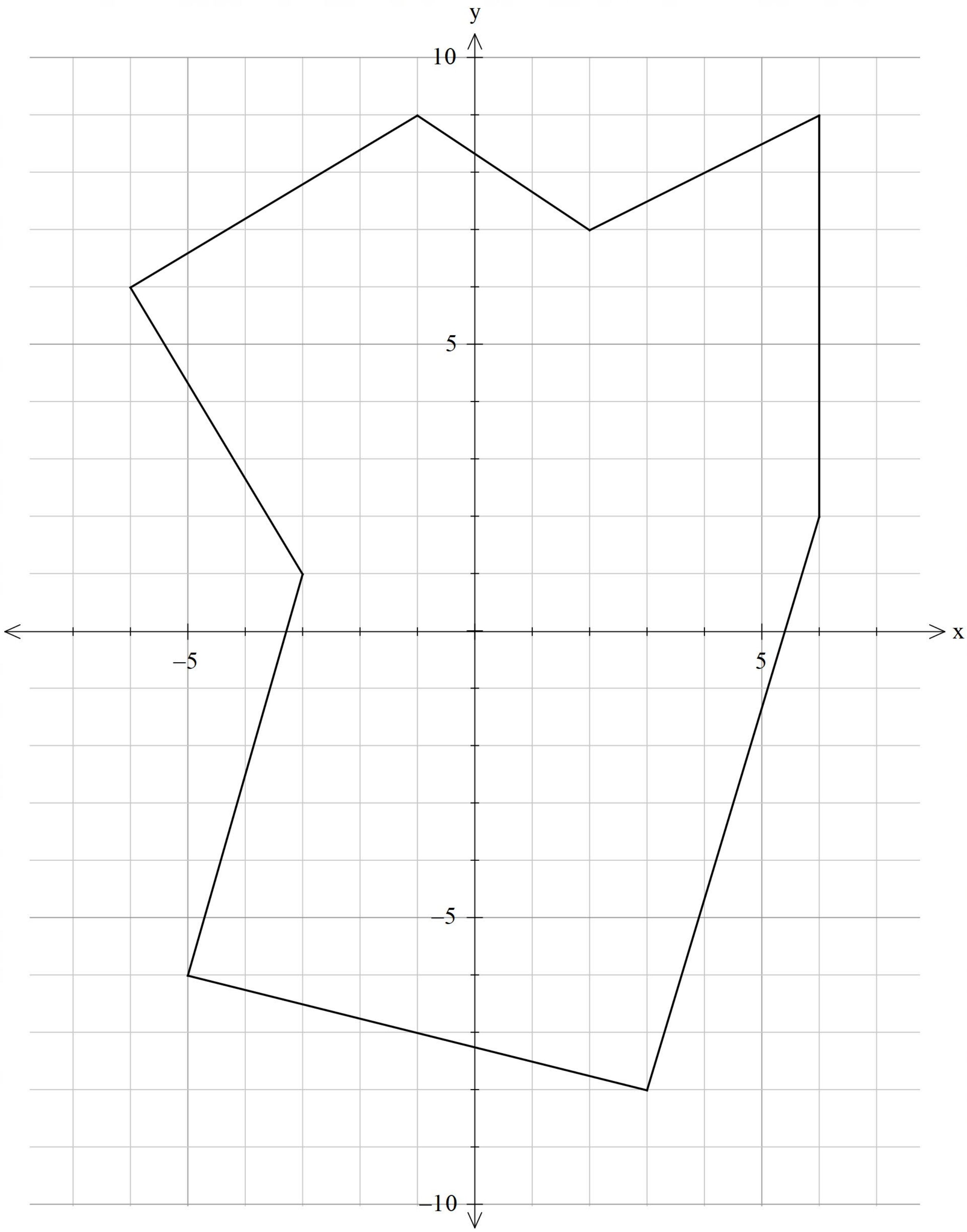

y
10
5
x
–5
5
–5
–10


y
10
5
x
–5
5
5
–10


y
10
5
x
–5
5
5
–10


y
10
5
–5
–10
–5
5
x


y
10
5
–5
–10
x
–5
5


y
10
5
–5
5
x
–5
–10

Sketch of shape	Coordinates of vertices	Estimate of centre of gravity

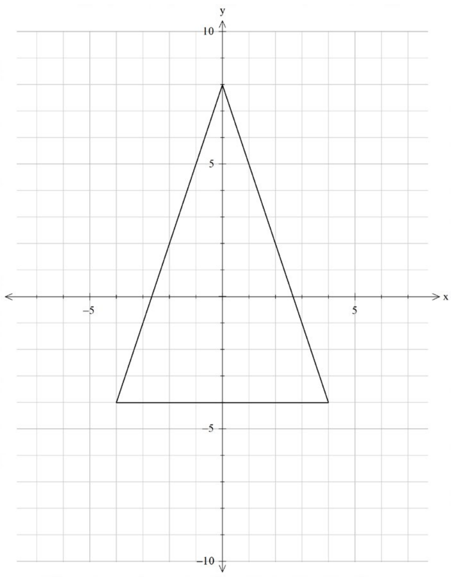
Appendix A.6 | Connected midpoints

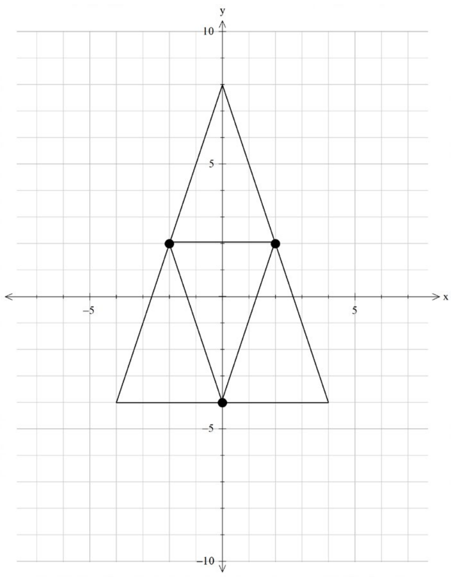
Lesson 3

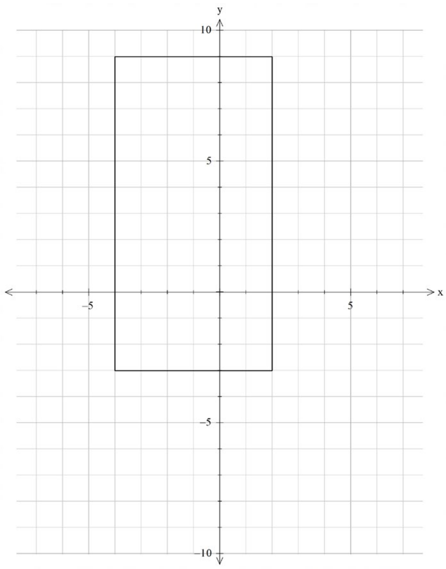
Instructions for teacher

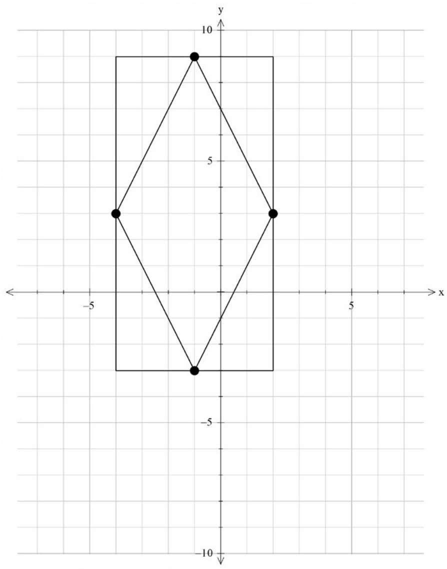
This learning activity looks at combining congruence, angle relationships, coordinate geometry and ratio skills to explore what happens when the midpoints of the adjacent edges of triangles and quadrilaterals are connected.

Students use a second copy of the triangles and quadrilaterals from Appendix A.5, cutting them out and calculating the midpoint of each line segment. Once they have calculated the midpoints, they connect the adjacent midpoints and explore the shapes which are created. This will produce figures such as those shown below.





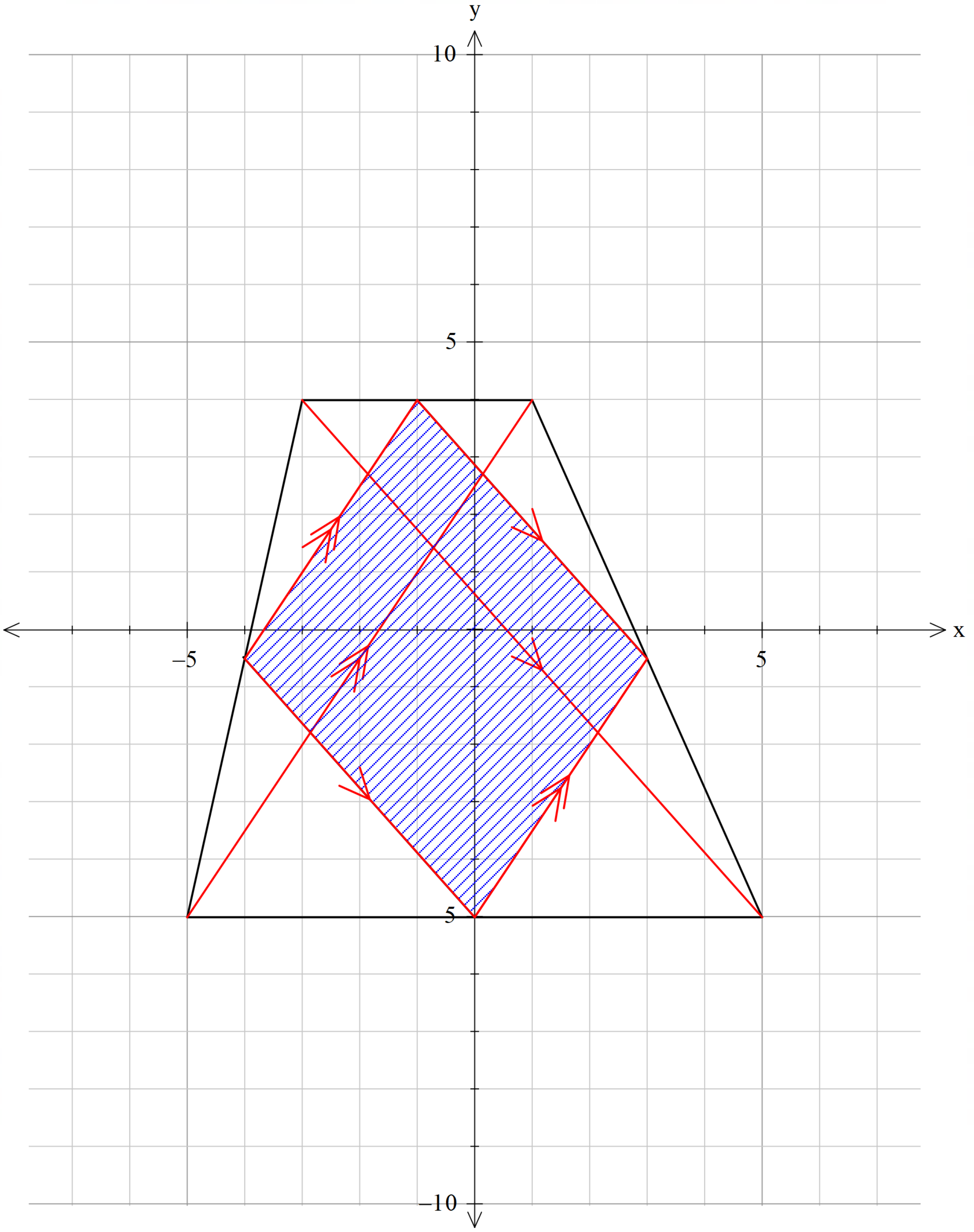




Behind the maths

When connecting the midpoints within a triangle, each new line drawn is parallel to the edge opposite. When all three are drawn, a series of alternate and corresponding angles are formed, and each line segment is the same length as those which it is parallel to. The 4 triangles which are produced are all congruent. Further, when folded up, these triangles will always form the net of a triangular pyramid.

When connecting the adjacent midpoints within a quadrilateral, each new line drawn is parallel to a perpendicular bisector. As there are two lines for each parallel bisector, the result is a quadrilateral every time.



Appendix A.7 | Regular pentagon and hexagon

Lesson 3

Instructions for teacher

This appendix provides a template for students to examine the patterns which occur when joining the adjacent midpoints of the edges of a pentagon. Depending on their coursework this year in Mathematics, they may be able to talk about the similar triangles formed and prove why they are similar. If they have not covered this, this activity provides an opportunity to explore the angles and ratios of the side lengths of the polygons formed.

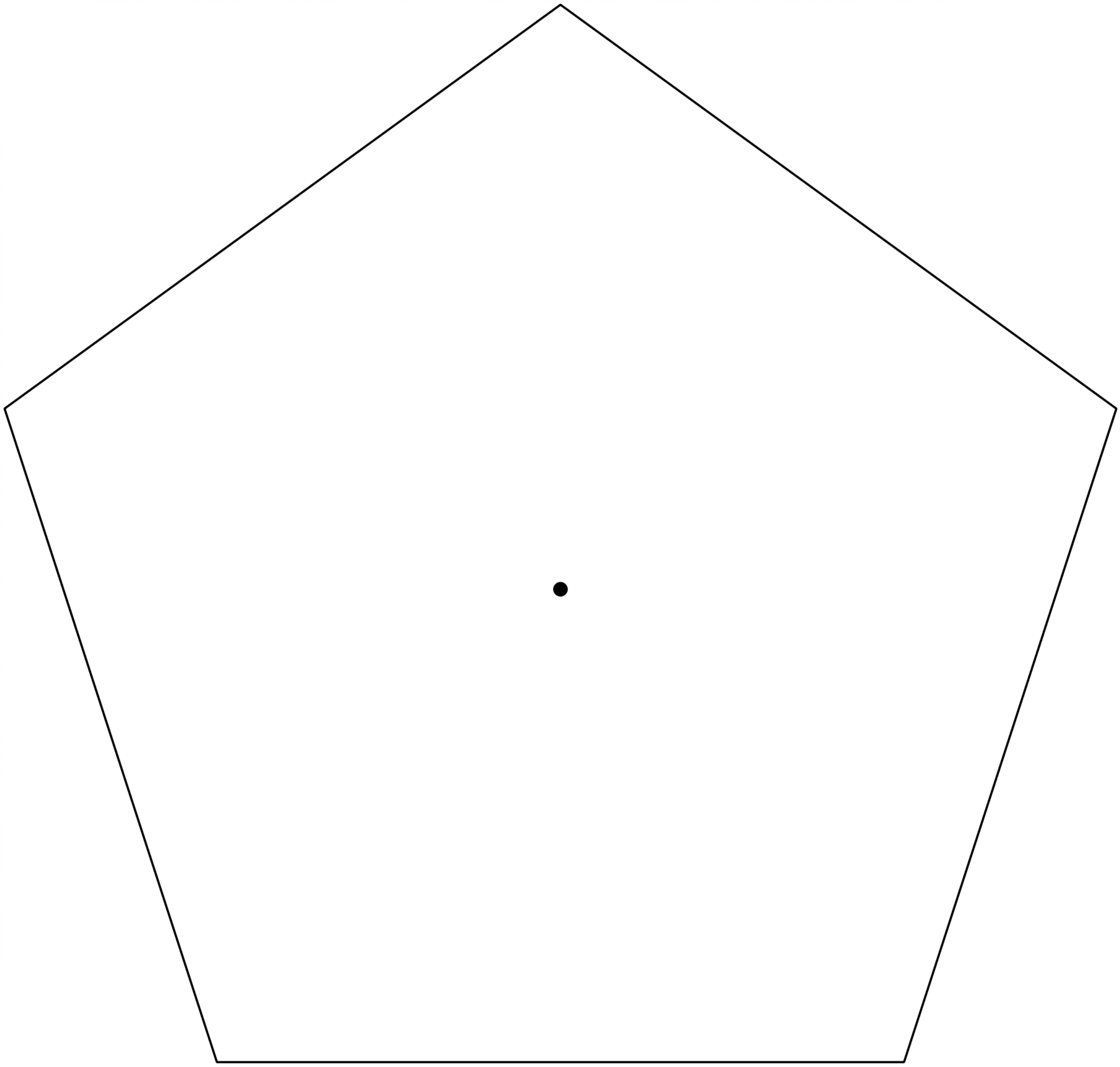
Extend students by looking at the ratio of the perimeter and area of the larger shape to the perimeter of each subsequent shape.

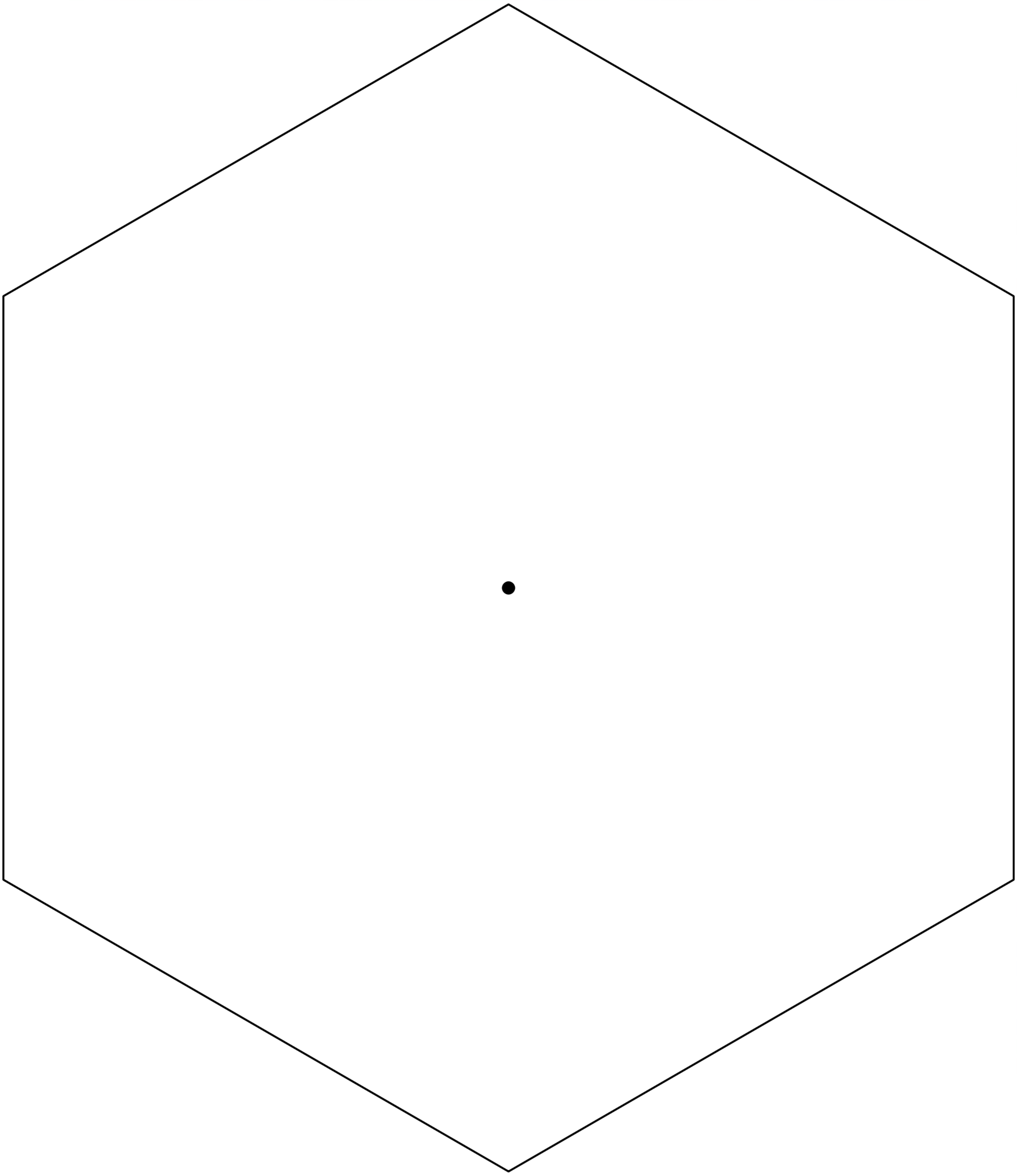
Learning task: Regular pentagons and hexagons

Your task is to connect the midpoint of the adjacent edges of the pentagon and hexagon on the following pages. Repeat this on the resulting shapes and comment on what you notice.

Explore the shapes that are made, comparing the angles, and the ratio of the side lengths.

- What do you notice about the perimeter of each shape that you make compared to the perimeter of the previous shape?
- What do you notice about the area of each shape that you make compared to the area of the previous shape?
- Is there a rule you could make to determine the perimeter or the area of the next shape?
- Is there a rule you could make to determine the perimeter or the area of any shape compared to the first shape?





Appendix A.8 | Pythagoras proof template

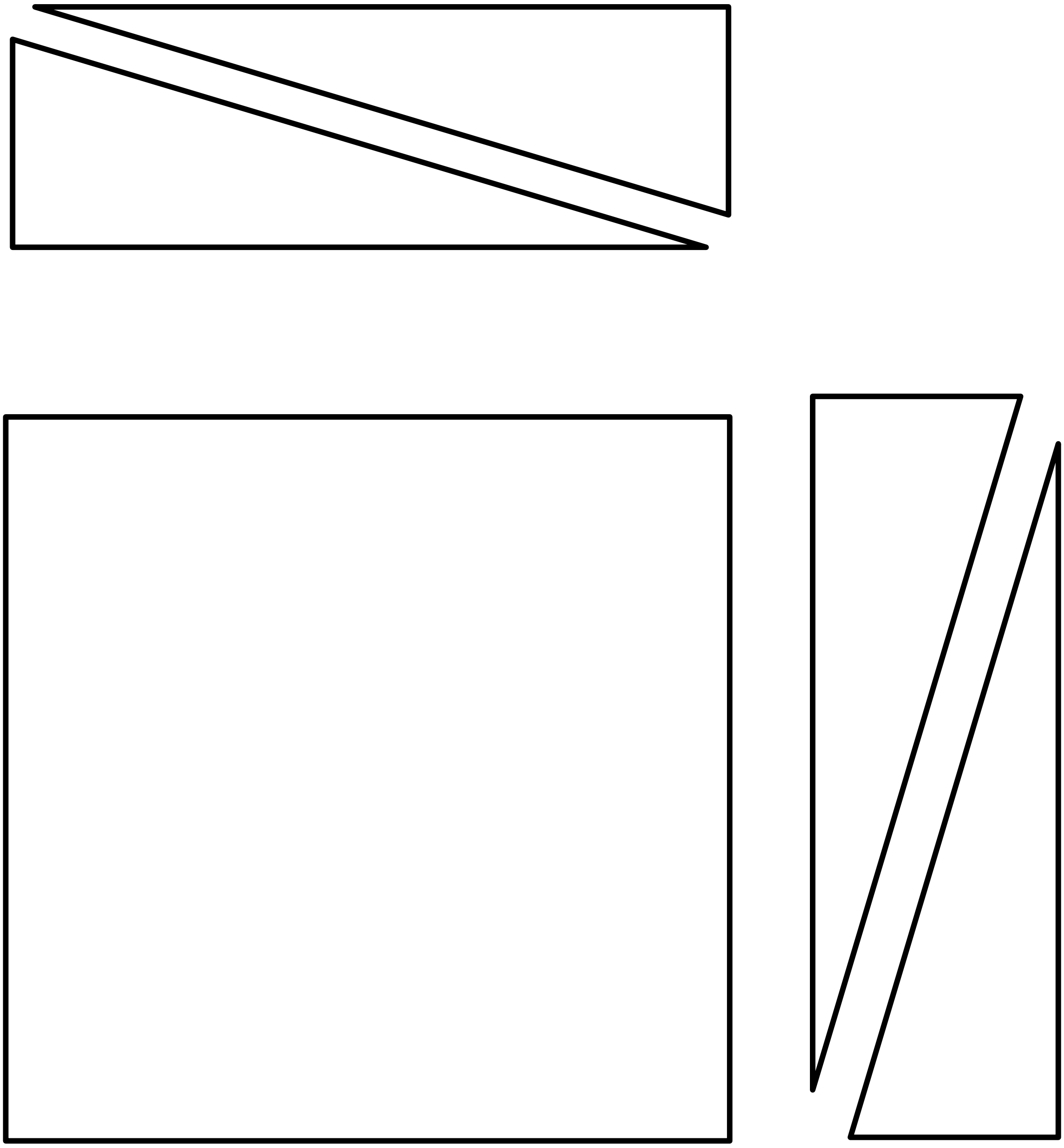
Lesson 5

Instructions for teacher

The following is one example of a physical representation of Pythagoras' Theorem. Many of these exist; determine the best possible representation for the classroom context.

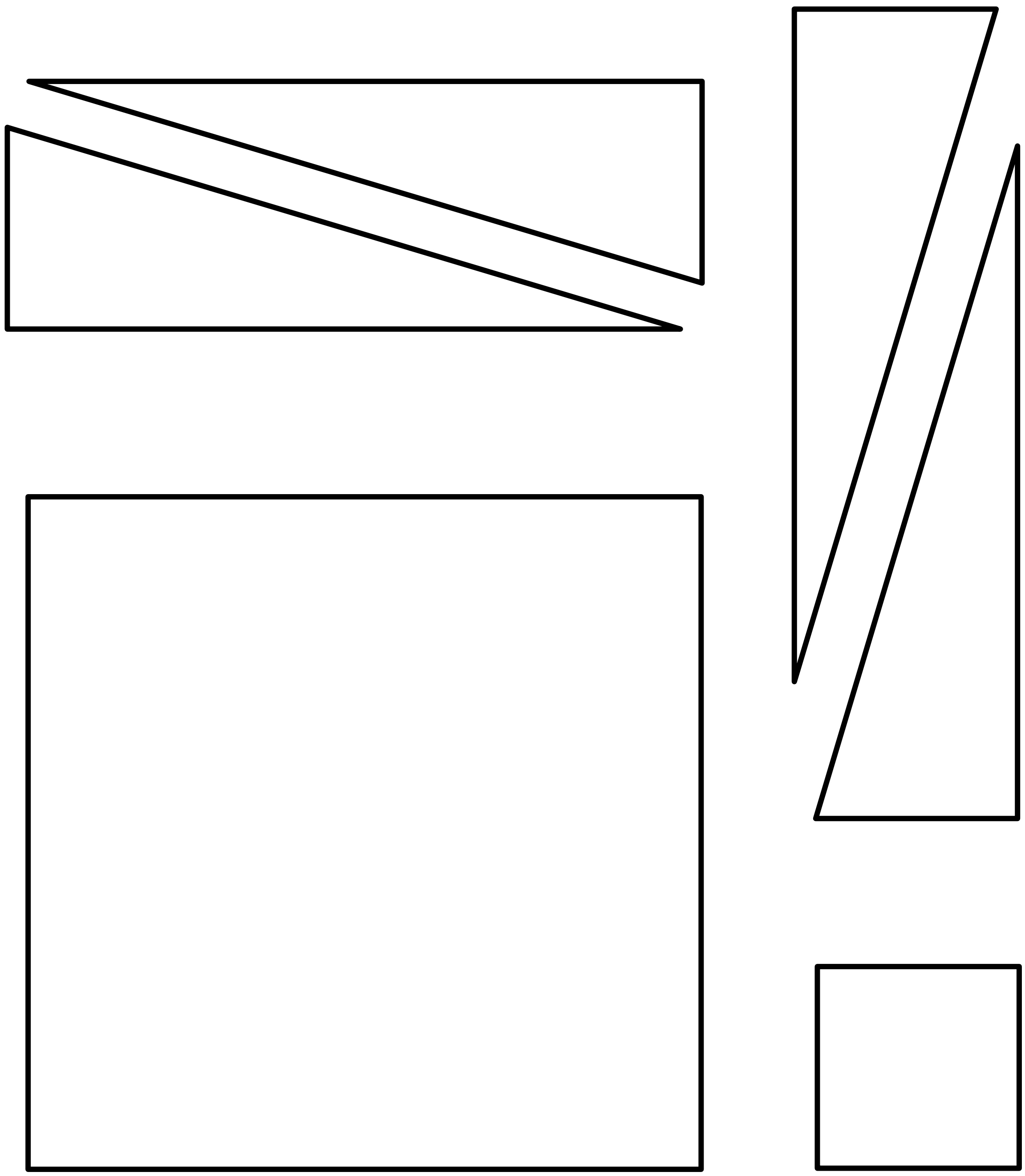
Learning task: Pythagoras proof template

Cut out the following and arrange to make a single square. In your book, describe the total area of your square.



Learning task: Pythagoras proof template

Cut out the following and arrange to make a single square. In your book, describe the total area of your square.



Appendix A.9 | Exit ticket template

Lesson 5

Instructions for teacher

The following is an example of a template to use as an exit ticket. Either put a specific question on them before you print them, put a question on the board or pose the question verbally, getting students to write their response on the appropriate section.

Once you have seen their response, gather this portion and leave them with the Exit ticket tab. This provides you with a good source of formative assessment to identify the achievement of each student in the lesson. Given that the template shows numerous tickets on one page, having three or four different questions will make sure that each individual student gets the chance to provide their own answer. Students may want to get support from their peers to promote conversations about what they need to do.

NAME: ________________________________ DATE: _________ CLASS: ______

Question: ____

EXIT TICKET

NAME: ________________________________ DATE: _________ CLASS: ______

Question: ____

EXIT TICKET

NAME: ________________________________ DATE: _________ CLASS: ______

Question: ____

EXIT TICKET

Appendix A.10 | Right-angled triangles

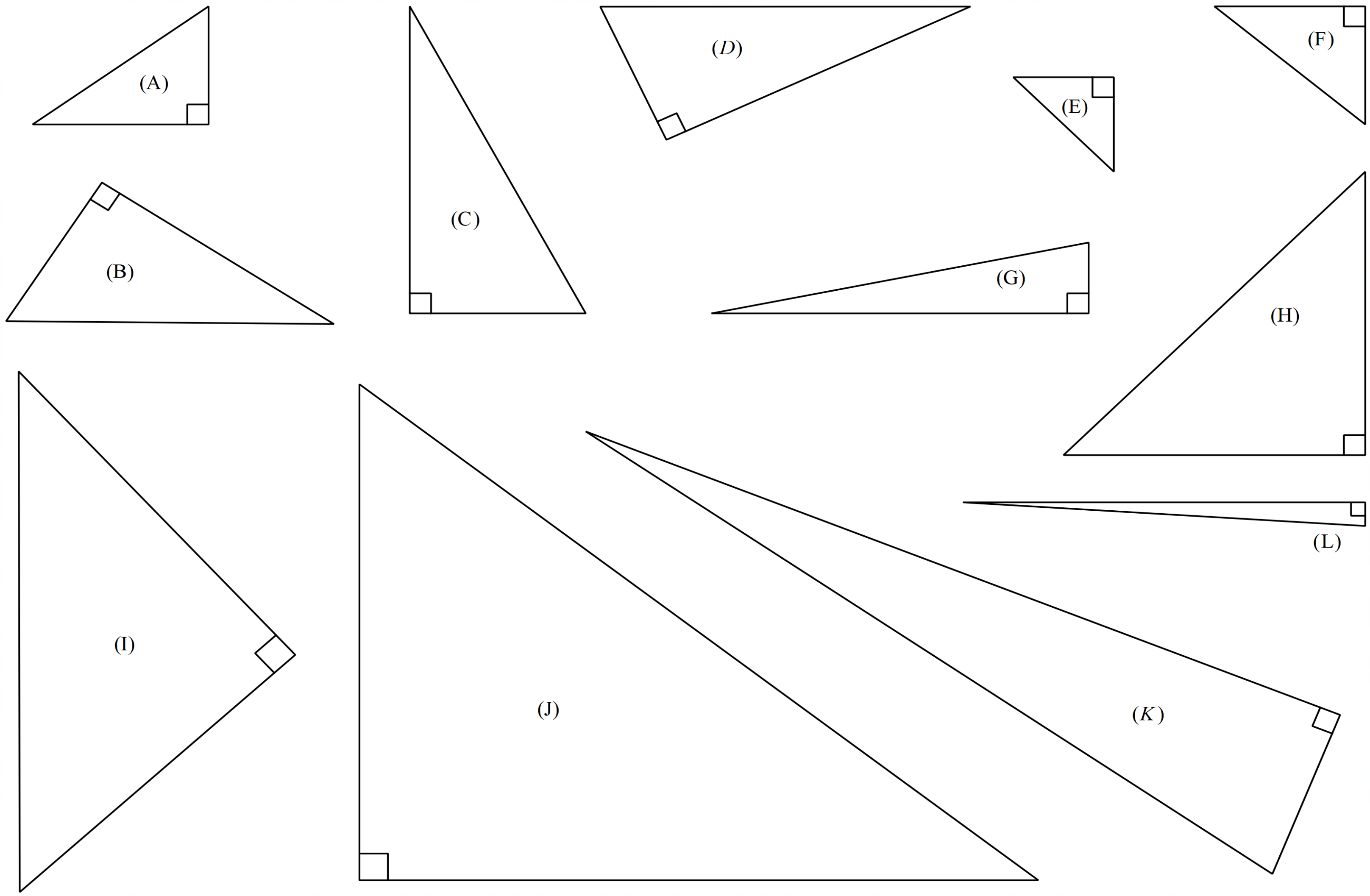
Lesson 5

Instructions for teacher

This activity involves students measuring the sides of right-angled triangles, comparing the squares of these measurements to develop their own version of Pythagoras' Theorem. Students are to work in mixed-ability groups (3–4 students) to discover the relationship $a^2 + b^2 = c^2$.

Learning task: Right-angled triangles

Measure the triangles to the nearest millimetre and record your measurements in the table. Use these measurements to complete the table below. Once you have completed the table, see if you can find a rule which relates the lengths of the sides of the triangle.



Triangle	Short leg (a)	Short leg (b)	Long leg (c)	a^2	b^2	c^2
A						
B						
C						
D						
E						
F						
G						
H						
I						
J						
K						
L						

Appendix A.11 | Ski resort planning

Lesson 8

Instructions for teacher

This open learning task allows for students to investigate gradient in the context of a ski resort. Students start by classifying ski slopes as easy, medium or hard, depending on the slope. From here, students develop a specific classification of slopes, comparing their length to their height. To look at positive and negative slopes, students will look at ski lifts compared to ski slopes.

After students have explored and defined the gradient in this context, they design their own ski resort, meeting certain classification of slopes.

This activity can be as directed or open as the classroom context allows.

If required, allocated more time to the teaching and learning sequence to fully explore this activity.

Images to rank and classify slopes

Provide each group of students with a copy of each of the images on the following pages.

SLOPE 1

SLOPE 2

SLOPE 3

SLOPE 4

SLOPE 5

SLOPE 6

SLOPE 7

SLOPE 8

SLOPE 9

SLOPE 10

Learning task: Ski resort planning

In your group of up to four students, you will work together to create your own ski resort. In order to do this, you will need to determine the slope of different ski runs. This is called the **gradient**. Follow the steps in each part to create your own ski resort with a range of exciting ski runs.

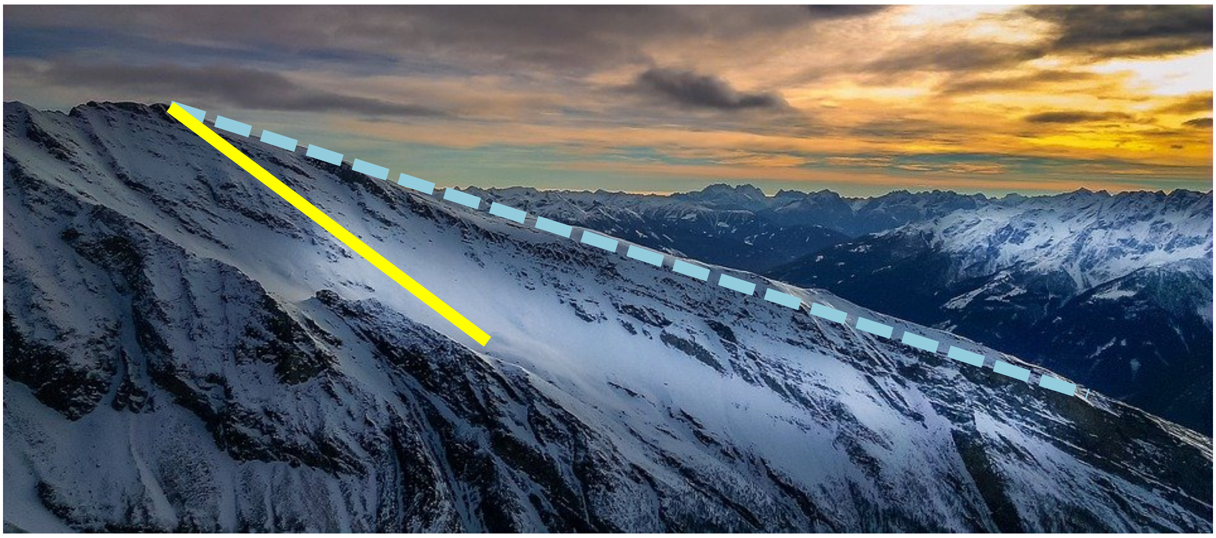
Part 1: Classifying slopes

Your teacher will provide your group with 10 images of ski slopes. Classify these as easy, medium and hard slopes.

- What are the key features of an easy slope?
- What are the key features of a hard slope?
- Is there a measurable feature which can be used to determine if a ski slope is easy, medium or hard?

Part 2: Features of a slope

Mathematicians like to be able to talk about things using numbers and measurements. In your group, determine all of the possible features that could be used to talk about the blue dashed ski slope shown below:



Rank the possible features of a slope in order of most influential/useful in determining the slope to the least influential/useful.

Compare the blue ski slope (broken line) to the yellow ski slope (solid line). Which slope is more difficult to ski down and why? Are you able to justify this difference with the features you have identified above? Are there any features you have rated high which are not useful in comparing the two slopes?

Part 3: Measuring the slopes

Draw a line on each picture that you think best represents this slope. Choose two points on the line you have drawn for each slope. Determine the vertical and horizontal distance between these points. You can measure to the nearest mm on your images. Put this information in the table below.

Slope	Vertical distance	Horizontal distance	Slope rank	Gradient (complete later)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

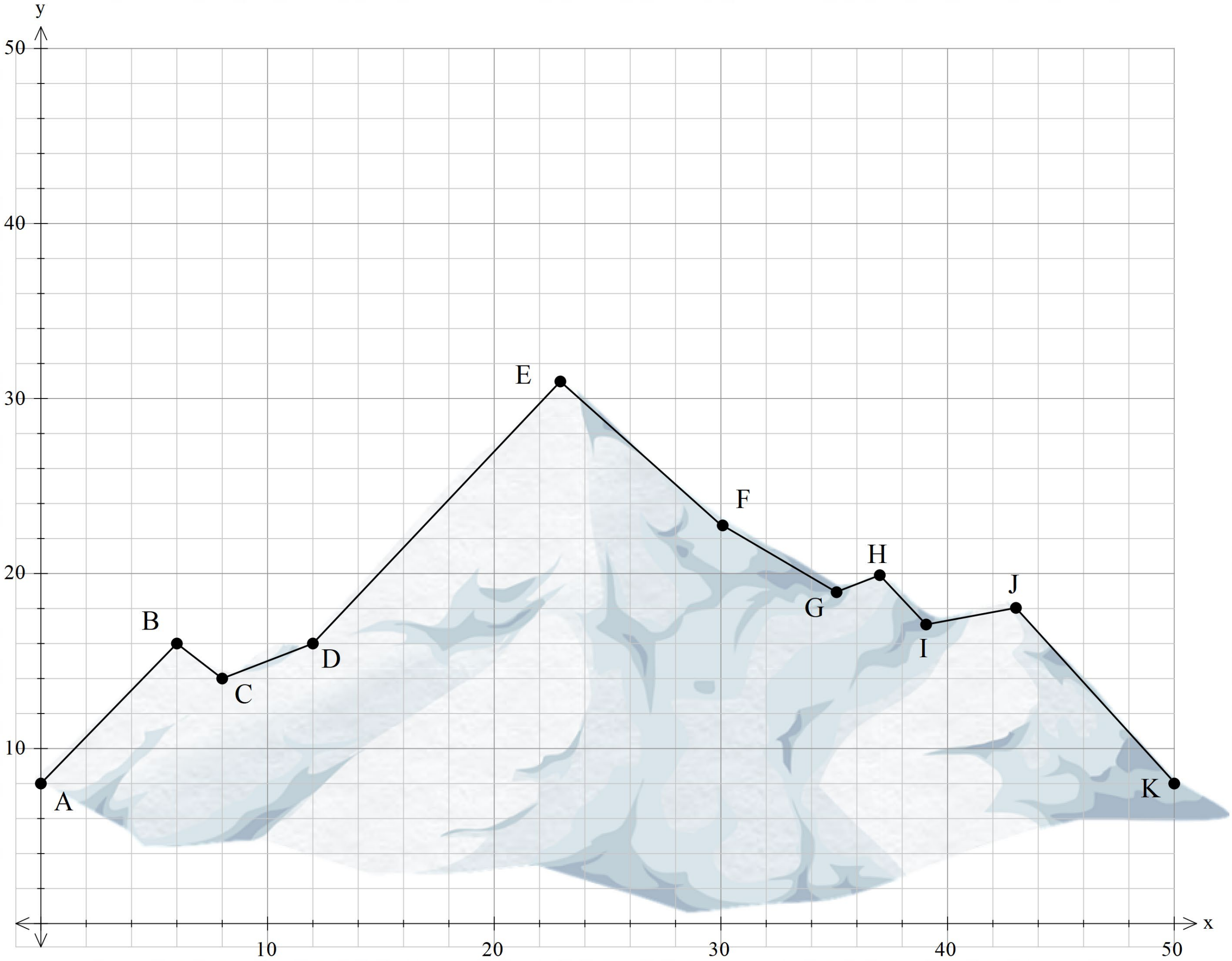
Do you think that you can get an accurate measurement of each of the slopes?

What extra information would make your measurements more accurate?

What might be stopping your measurements from being as accurate as possible?

Part 4: A model of a ski slope

Below is a two-dimensional cross-section of a ski slope during summer. Each of the 10 slopes is shown as a straight line on the diagram. Determine the coordinates of each point, indicated by the letters, A to K. Record your results in the table below. Rank these slopes according to their steepness. Leave the gradient blank for now.



	A	B	C	D	E	F	G	H	I	J	K
Coordinates											

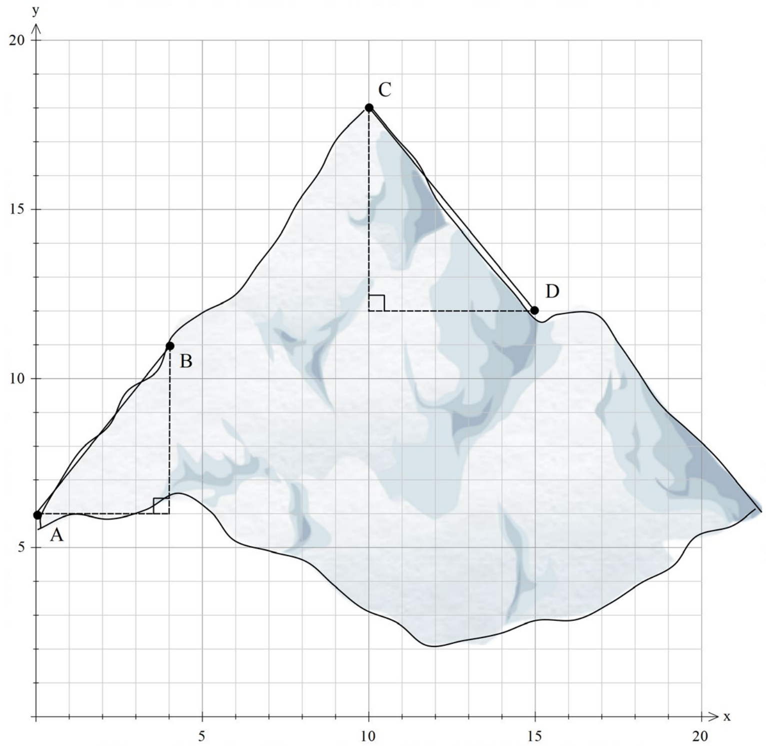
	AB	BC	CD	DE	EF	FG	GH	HI	IJ	JK
Slope rank										
Gradient										

Part 5: The gradient

In Mathematics, the gradient is the ratio of the vertical change compared to the horizontal change. It is expressed as a fraction:

$$\left[\text{gradient} = \frac{\text{change in vertical distance}}{\text{change in horizontal distance}}\right] \text{ or } \left[\text{m} = \frac{\text{rise}}{\text{run}}\right]$$

When this fraction is evaluated, it determines the steepness of a slope. Some examples are shown below.



Gradient $\overline{\text{AB}}$ $= \frac{\text{change in vertical distance}}{\text{change in horizontal distance}}$

$= \frac{3}{2} = 1.5$

Note: this slope is going up when moving from left to right. This means the slope is positive.

Gradient $\overline{\text{CD}}$ $= \frac{\text{change in vertical distance}}{\text{change in horizontal distance}}$

$= -\frac{3}{2} = -1.5$

Note: this slope is going down when moving from left to right. This means the slope is negative.

When you move from left to right, if you need a ski-lift to go up, it's positive; if you can ski down, it's negative.

Try it yourself.

Calculate the gradient for each of your slopes in Part 3 and Part 4.
Remember to check if they are positive or negative.

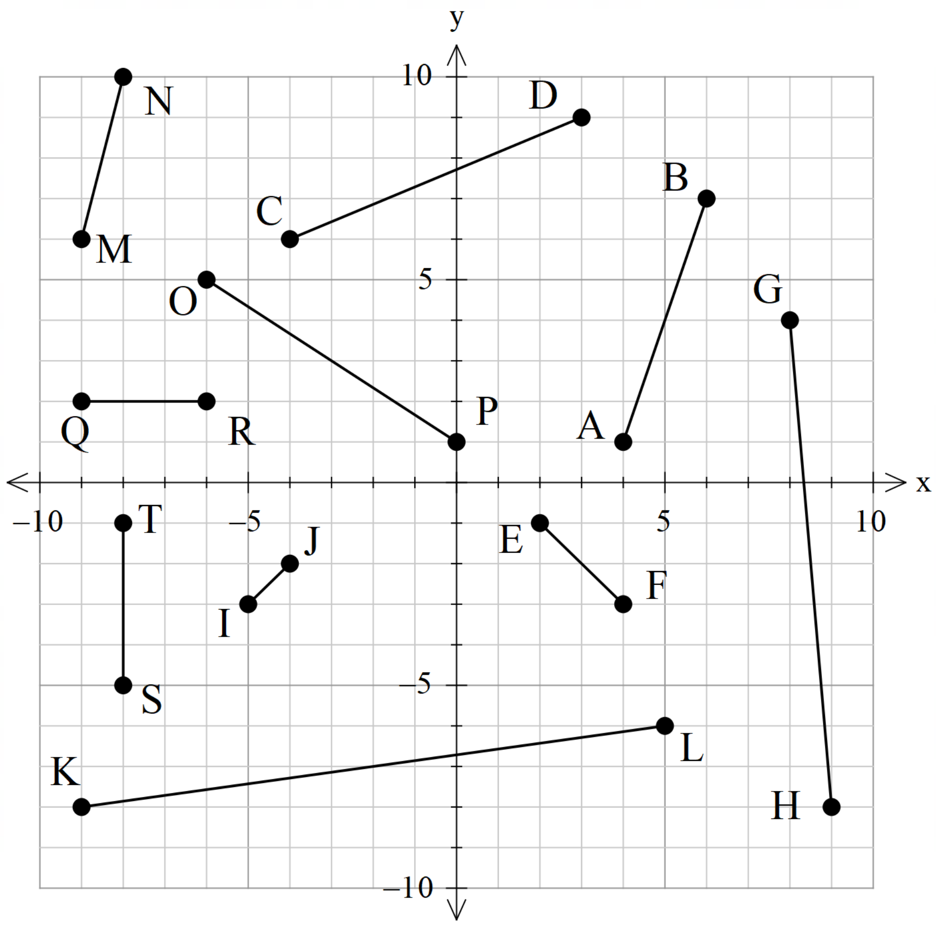
Part 6: Design your own ski slope

Use the Cartesian plane on the following page to design your own ski slope, like those shown in Parts 4 and 5. You will need to consider the following points.

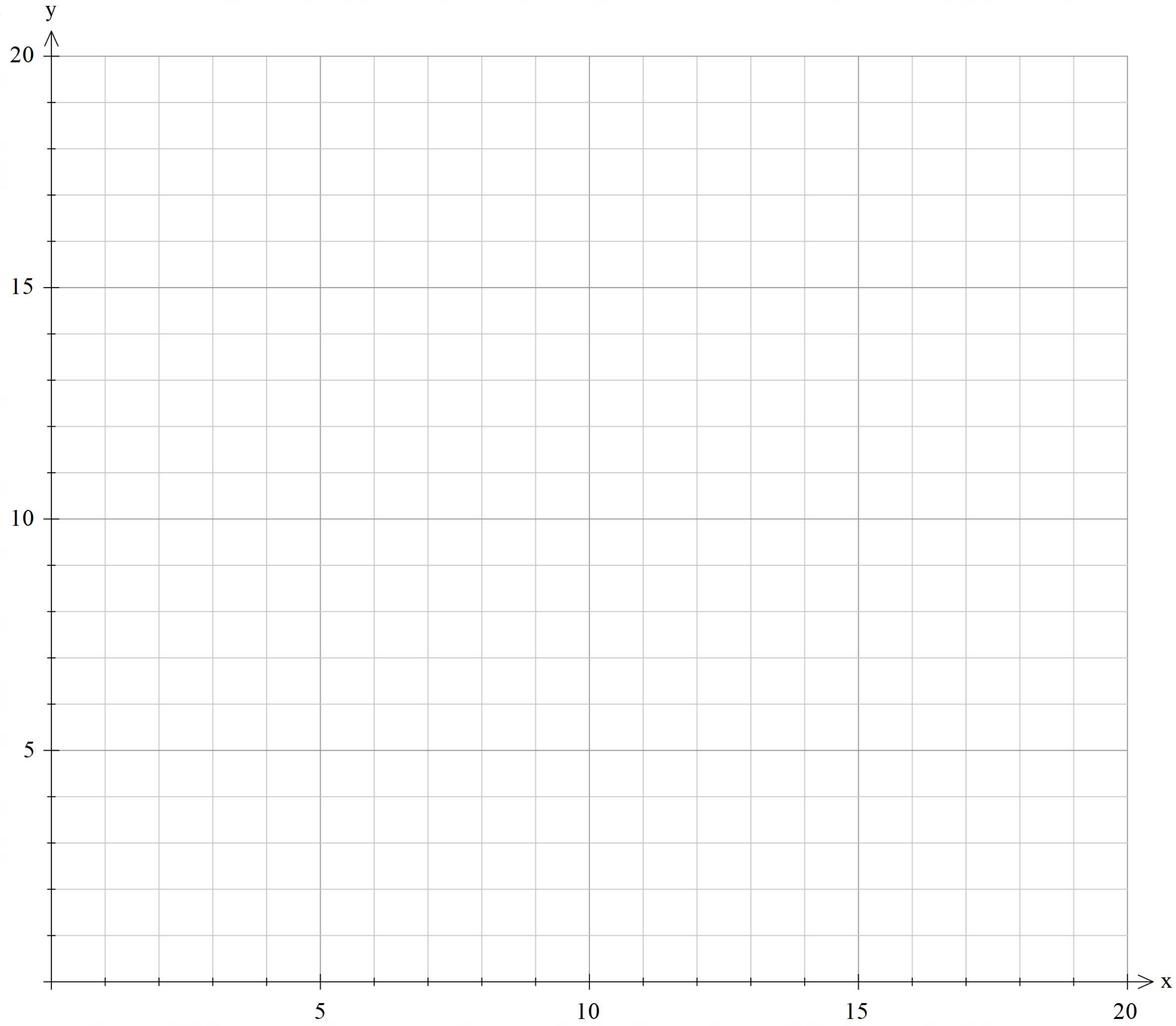
- You must have at least one easy, one medium and one hard slope. An easy slope has a gradient between 0 and 1, a medium slope has a gradient of exactly one and a hard slope is greater than one.
- You must have appropriate ski lifts to each slope. These cannot have a gradient greater than 2.
- You must have one flat section for cross-country skiing. What is the gradient of this section?
- Once you have designed your ski slope, label the start and end point of each slope, calculate the gradient for each of these slopes and label it on the diagram.
- When this is completed, determine the length of each slope.

Part 7: Calculations

Once you have designed your ski slope and labelled all of the points, determine the gradient of the line segments below.



	Gradient
AB	
CD	
EF	
GH	
IJ	
KL	
MN	
OP	
QR	
ST	



Space for working out

Appendix A.12 | Capital coordinates

Lesson 9: The distance formula

Instructions for teacher

Students use a map with an approximate scale to determine the straight line distance between each Australian capital city. Students determine the approximate coordinates of each capital city, determine the horizontal and vertical distances between these points and then determine the straight line distance between these points.

This activity is intended to help students develop their understanding of Pythagoras' Theorem into the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Question students to develop this formula, looking at how to calculate the horizontal or vertical distance using the coordinates.

Learning task: Capital coordinates

Use the map of Australia, Pythagoras' Theorem and the approximate locations of the capital cities to determine the straight-line distance between each city.

Coordinates:

Perth		Darwin		Hobart	
Adelaide		Canberra		Brisbane	
Melbourne		Sydney			

Determine the distance between each city in your book. Each unit on the map represents approximately 50 km in real life. Put your answers in the white boxes and the actual distances in the shaded boxes of the table below.

	Perth	Darwin	Adelaide	Melbourne	Canberra	Sydney	Hobart	Brisbane
Perth	-							
Darwin		-						
Adelaide			-					
Melbourne				-				
Canberra					-			
Sydney						-		
Hobart							-	
Brisbane								-

What equation could you use to go straight from the coordinates of each location to the distance between the two points?

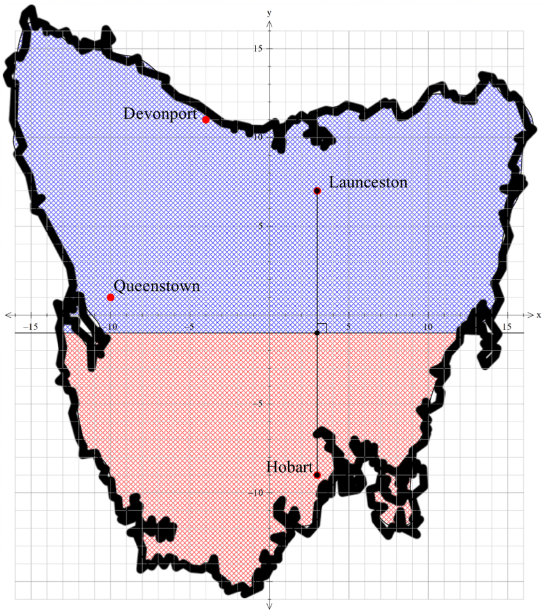
Appendix A.13 | Changing state lines

Lesson 9

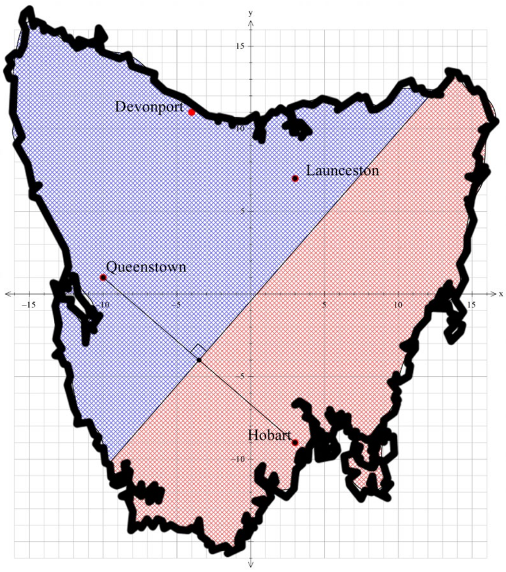
Instructions for teacher

Students split a state or country into regions where the capital city is the closest capital city geographically. A good example of this is the far northern region of Western Australia, which is five or six times closer to Darwin than Perth.

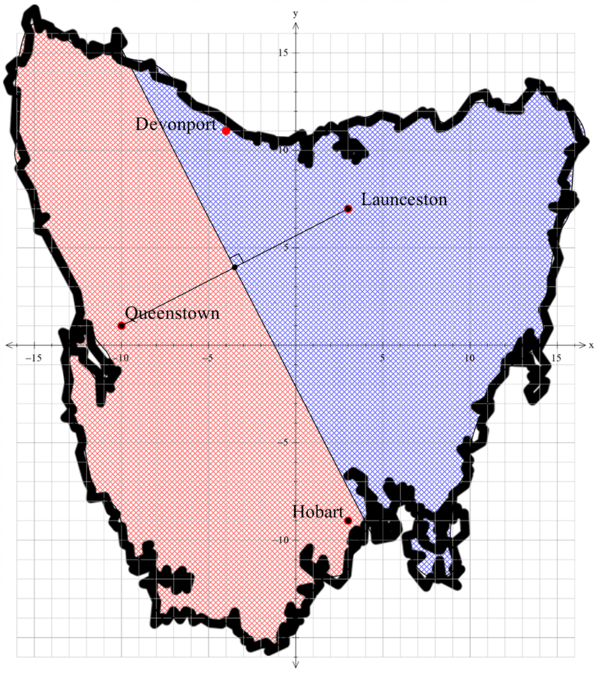
This process can be introduced by looking at a smaller area, such as a map of Tasmania and splitting it into two smaller regions, where Hobart and Launceston are the capitals. Students may need assistance to determine that they identify the line between the two cities and then draw a perpendicular line through the midpoint. Students verify this by calculating the distance of a point which is just above or just below the line they have created.



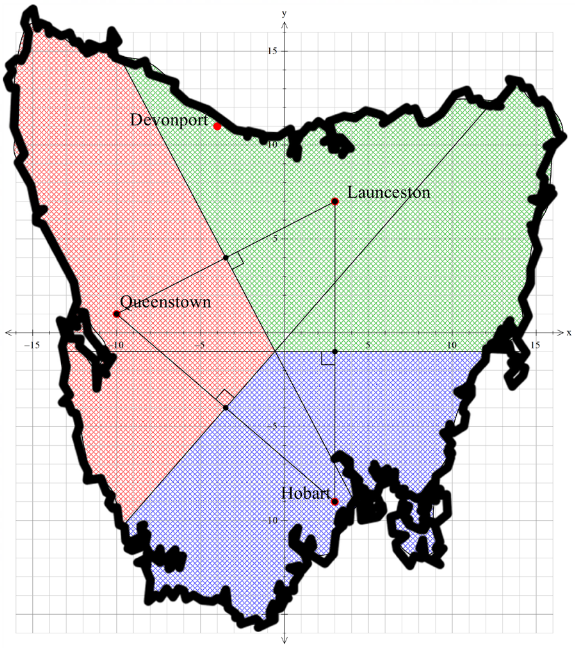
Once they have split Hobart and Launceston, they change the capitals to Hobart and Queenstown and see if this changes the shape of the regions.



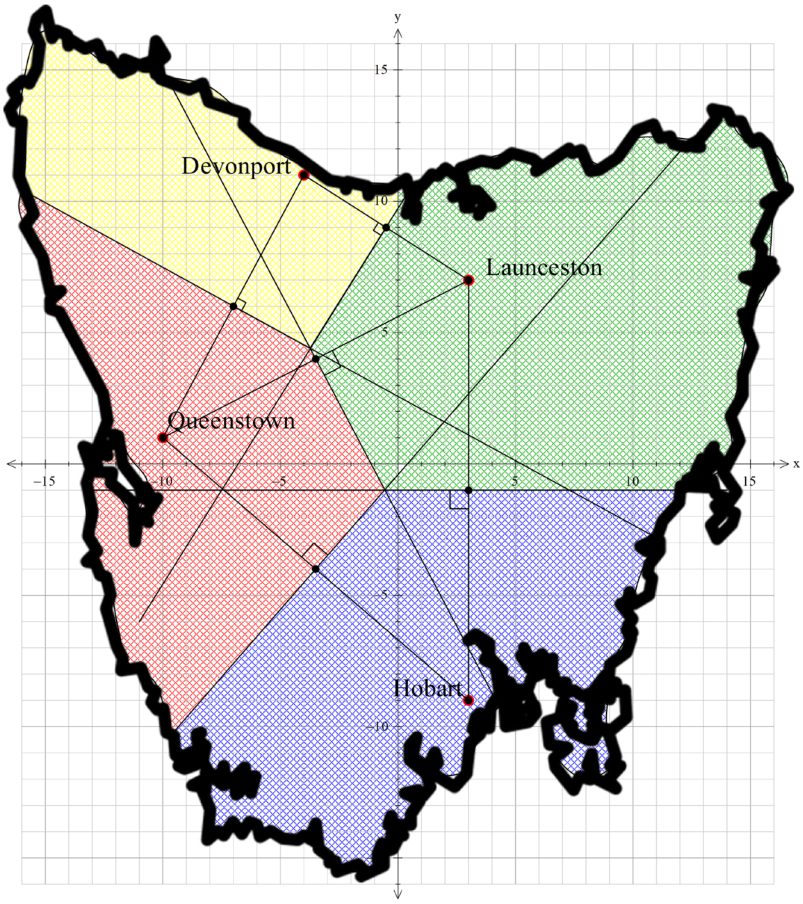
Repeat again with Launceston and Queenstown.

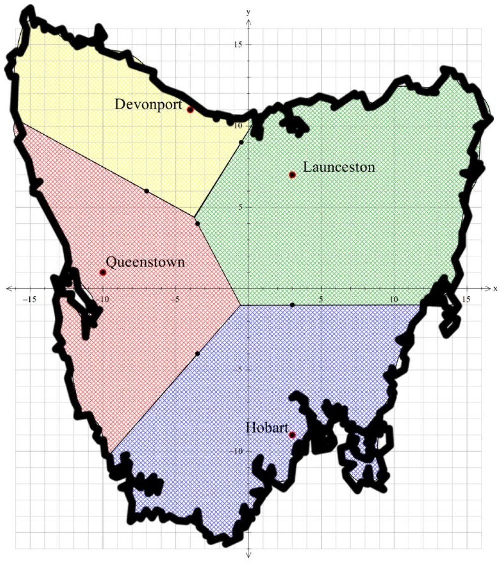


Ask the question: what would Tasmania look like if it were split into three regions where everywhere in each region was closest to the relevant capital? To aid students, have them look at the point where the lines perpendicular to the midpoint intersect. This represents the circumcentre of the triangle – the point each corner is equidistant from. To split it into appropriate states from here, connect the perpendicular lines. This process is shown below.



For students to add other cities as capitals, they need to draw the triangle which includes that city and repeat the process. Below is an example of Devonport as a capital city.





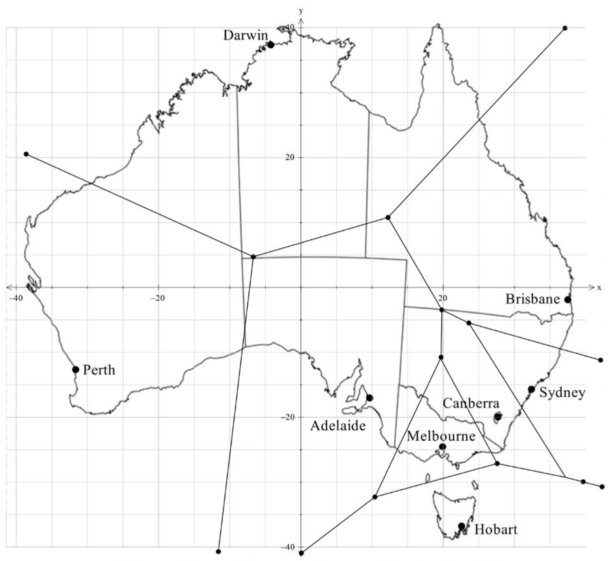
If students are able to produce this diagram, set a challenge to create new state lines for all of Australia. Use the Australia map template from Appendix A.12.

An example of what it could look like can be found at https://i.imgur.com/GMVV5JO.png.

The interactive applet at http://cfbrasz.github.io/Voronoi.html can be used to complete this activity.

Another version of interpreting how these diagrams are made can be found on the Khan Academy website:

- *Voronoi partition* [video] https://www.khanacademy.org/computing/pixar/pattern/dino/v/patterns2_new.



Learning Task: Changing state lines

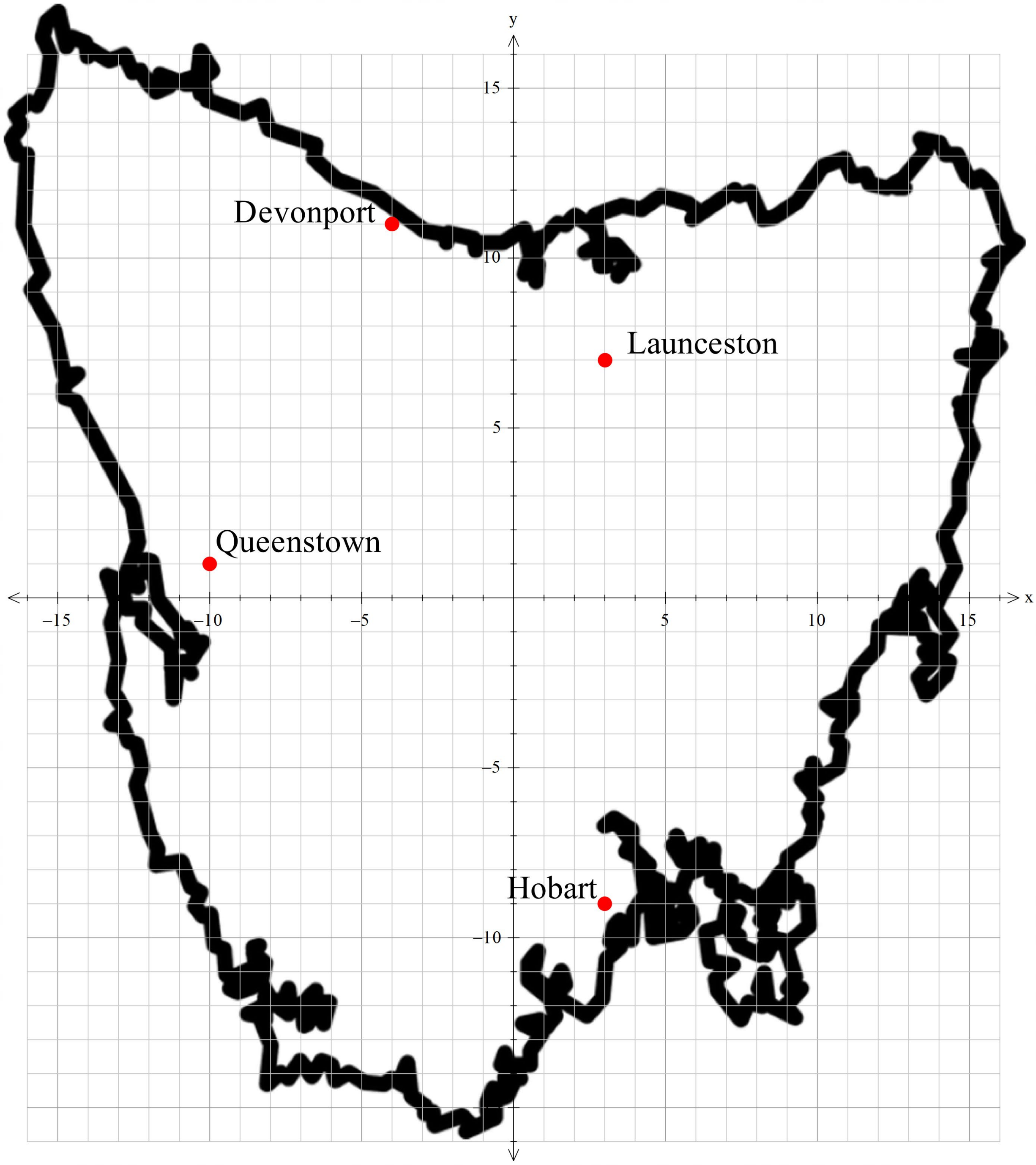
On the map of Tasmania below, draw lines to split it into four areas so that every point within each area is closest to the main city Devonport, Launceston, Queenstown or Hobart. You will need to use your knowledge of midpoint and distance between two points to check your answer.

Before you start with all four cities, start with two cities and then three cities. If you get stuck or need help, ask your teacher or watch the following video:

- *A mathematical guide to social distancing*
  https://www.youtube.com/watch?v=ImbegJm4EpA

Alternatively, explore the interactive found at Git Hub

- Voronoi diagram generator
  http://cfbrasz.github.io/Voronoi.html.



Appendix A.14 | Cartesian plane template

Lesson 10

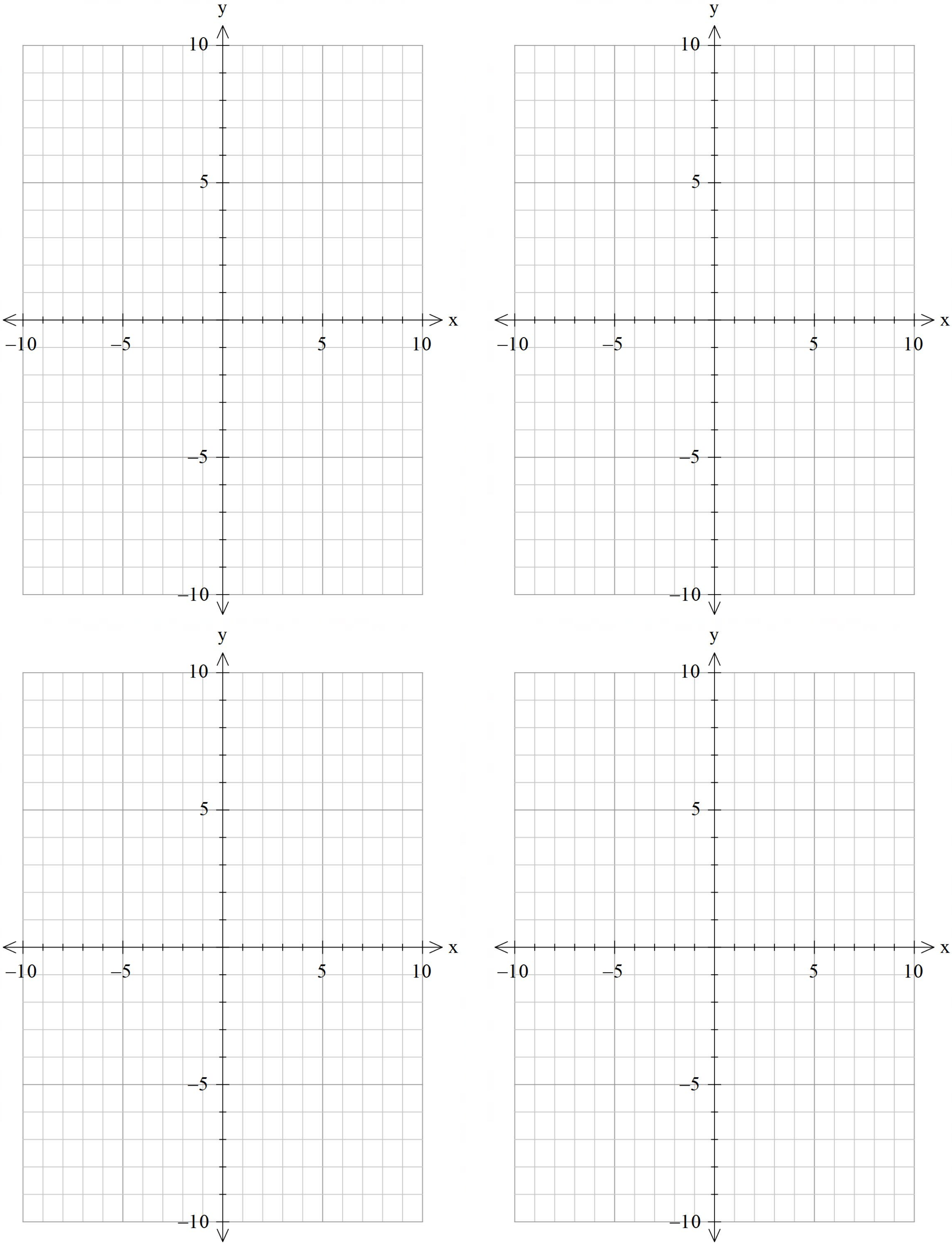
Instructions for teacher

This learning activity allows students to create their own questions. Organise students into groups of four. Each student starts by drawing two sets of coordinates on the top left Cartesian plane, then passing the diagram to the student on their left. This student determines the midpoint, the gradient, and the distance between these two points. They add two more pairs of coordinates to the next Cartesian plane and pass it to the left again. Each student will determine the midpoint, gradient and distance between the two points for each graph, so will do one, then two, then three and then four problems.

When all students have solved 10 problems, the group will compare their answers. Where there is a discrepancy, students work together to determine the correct approach.

Learning task

Using the Cartesian planes on the next page, your group will create a bank of practice questions. Starting with the top left Cartesian plane, add two points, labelling the coordinates. Pass this to the person on your left. This person is to determine the midpoint, the gradient and the distance between these two points. **They will show their working in their own book.** Once they have calculated this, they will add two points to the next Cartesian plane and pass it to the left again. This person will now determine the midpoint, the gradient and the distance between these two points for every Cartesian plane on the page. Repeat the process until every page is full of problems and each student has solved at least 10 problems.



y
10
5
x
−10
−5
5
10
−5
−10

APPENDIX B:
FORMATIVE ASSESSMENT TASK

Pythagoras and Beyond

Appendix B | Formative assessment task

Title of task

Pythagoras and Beyond

Task details

Description of task	Students test whether Pythagoras' Theorem holds true when using other shapes beyond the square formed on the edges of a right triangle.
Type of assessment	Formative assessment
Purpose of assessment	To inform the classroom teacher of the achievement of students to date in the unit and provide the students with an opportunity to demonstrate their understanding of Pythagoras' Theorem and coordinate geometry
Assessment strategy	Individual formative assessment in test conditions. Students can receive support or guidance from teacher, but note this on their work.
Evidence to be collected	Individual student workbook
Suggested time	Up to 1 lesson in class

Content description

Content from the Western Australian curriculum

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Task preparation

Prior learning

Students are six lessons into their unit on coordinate geometry, including midpoint and distance between two points, and Pythagoras' Theorem.

Students have determined the area of shapes, such as triangles, circles and rectangles, in previous years.

Assessment differentiation

Teachers should differentiate their teaching and assessment to meet the specific learning needs of their students, based on their level of readiness to learn and their need to be challenged.
Where appropriate, teachers may either scaffold or extend the scope of the assessment tasks.

Assessment conditions

- One-to-one interview may be used at any time in the assessment process to record anecdotal evidence and to clarify student understanding.

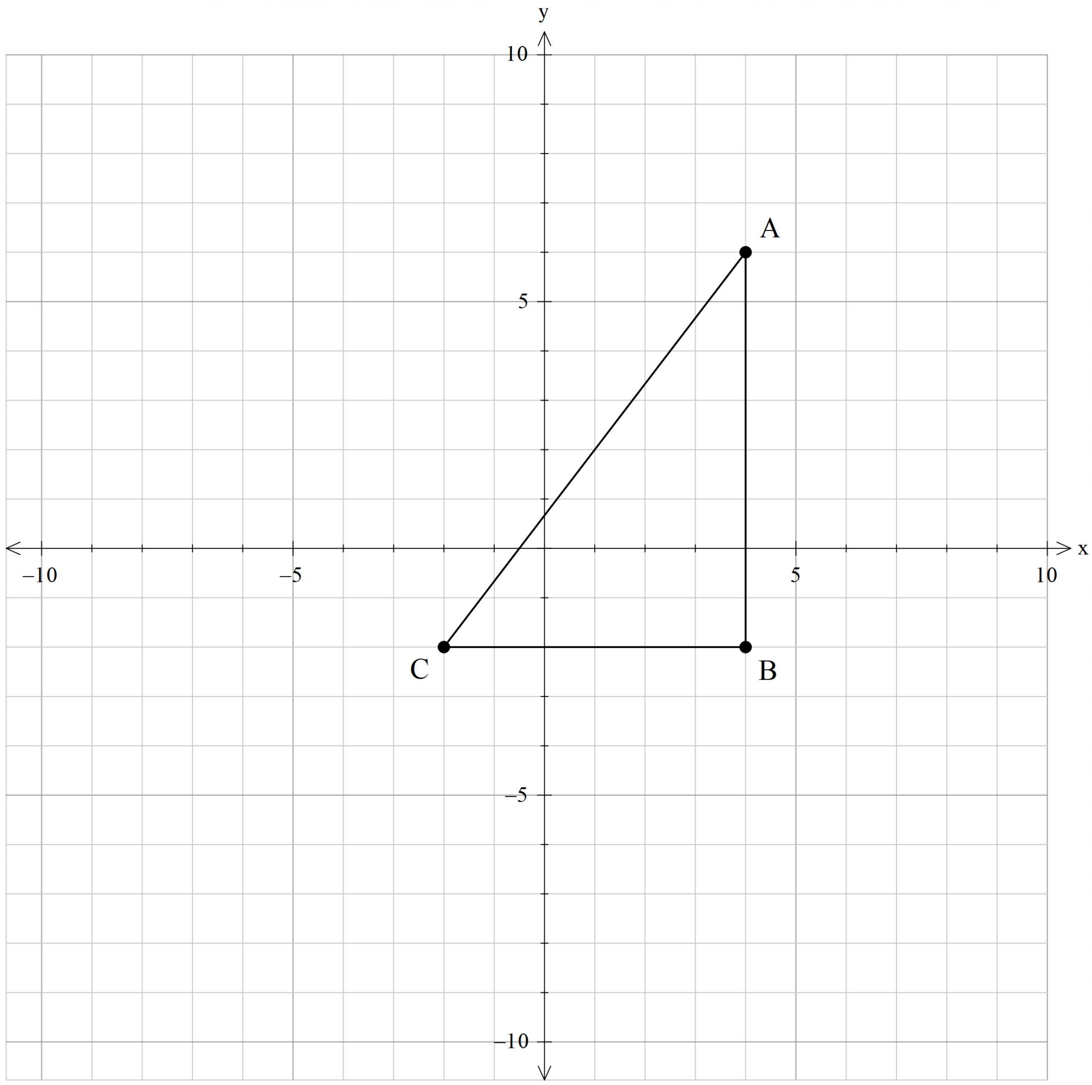
Resources

- calculator, ruler
- The following resources can be used to support students who have not demonstrated the expected Standard early in the task.
  - Pythagorean Theorem – Kassie Smith – University of Georgia http://jwilson.coe.uga.edu/EMAT6680Fa2012/Smith/6690/pythagorean%20theorem/KLS_Pythagorean_Theorem.html
  - Pythagoras Theorem using other shapes (Ep. 2) [video] – Maths advice on your device https://www.youtube.com/watch?v=6rCdvPI40R4
  - Surprising uses of the Pythagorean Theorem – Better Explained https://betterexplained.com/articles/surprising-uses-of-the-pythagorean-theorem/

Pythagoras and beyond – Task sheet

On the following pages, you will find a series of diagrams involving right-angled triangles. You will be determining the areas of the shapes which are made using the edges of these triangles and compare them using your knowledge of Pythagoras' Theorem.

1. Triangle ABC is a right-angled triangle. **(16 marks)**



a) Determine the lengths AB and BC. Use these to determine the length of AC. (4 marks)

____

____

____

____

____

____

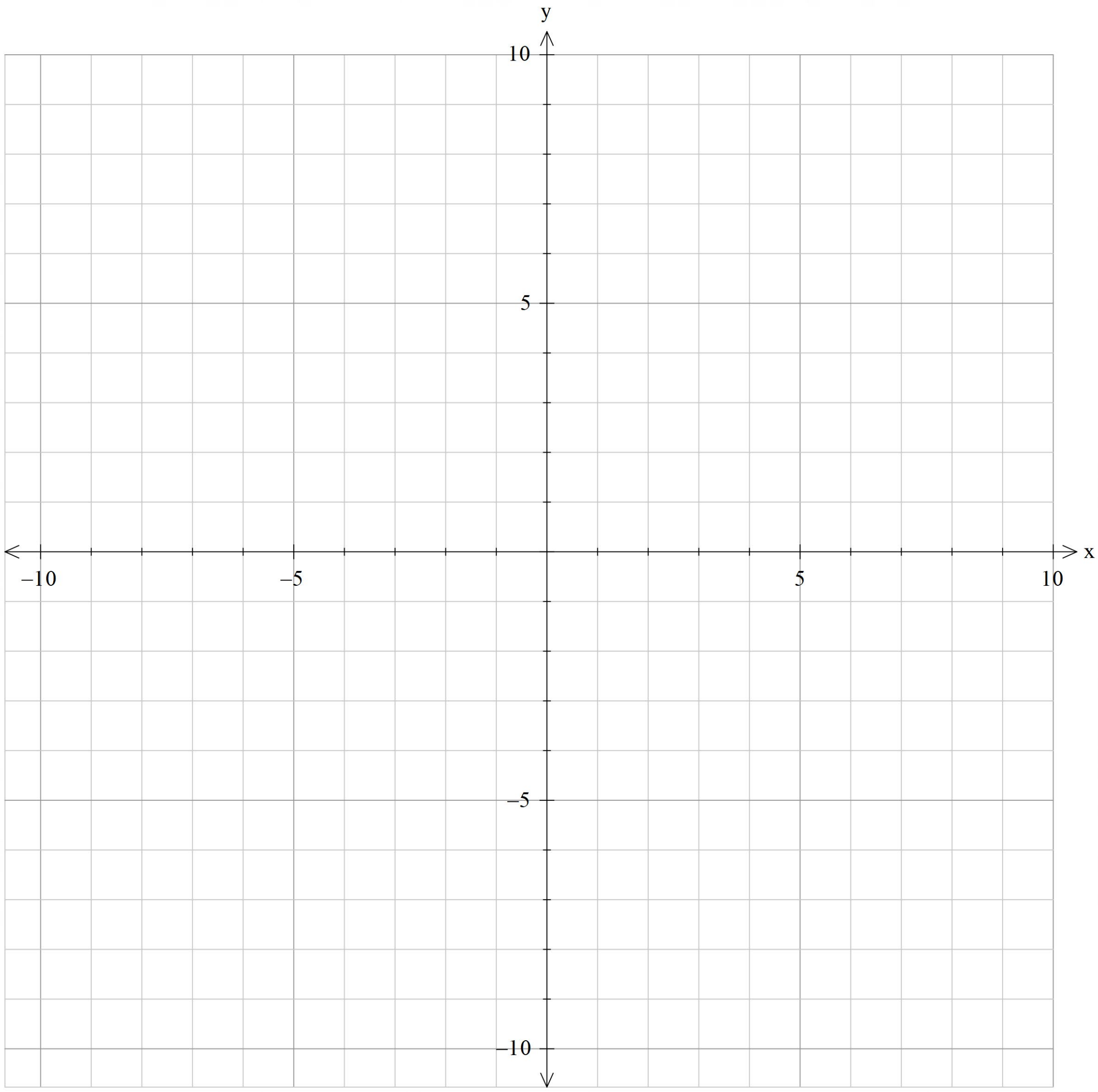
b) Determine the midpoint of each of the line segments, AB, BC and AC and mark these D, E and F respectively on the diagram above. Show your working out in the space provided. (4 marks)

c) Draw a square using line AD as the left hand edge.
Draw a second square using line BD as the left hand edge.
Repeat this process, drawing two squares along each edge.
If you need help for the slanted square, use the point $(-6,1)$ to help you.

Calculate the area of each of the rectangles you have formed.
What do you notice about the relationship between the areas? (8 marks)

2. Follow the steps to complete the diagram, then answer the questions. **(12 marks)**

a) Draw the triangle XYZ using the points X$(-4,-2)$, Y$(-4,4)$ and Z $(0,4)$.
Label the midpoint of each of these lines, L, M and N. (3 marks)



b) Determine the lengths of XY and YZ. Use these to determine the length of XZ. (4 marks)

____

____

____

____

____

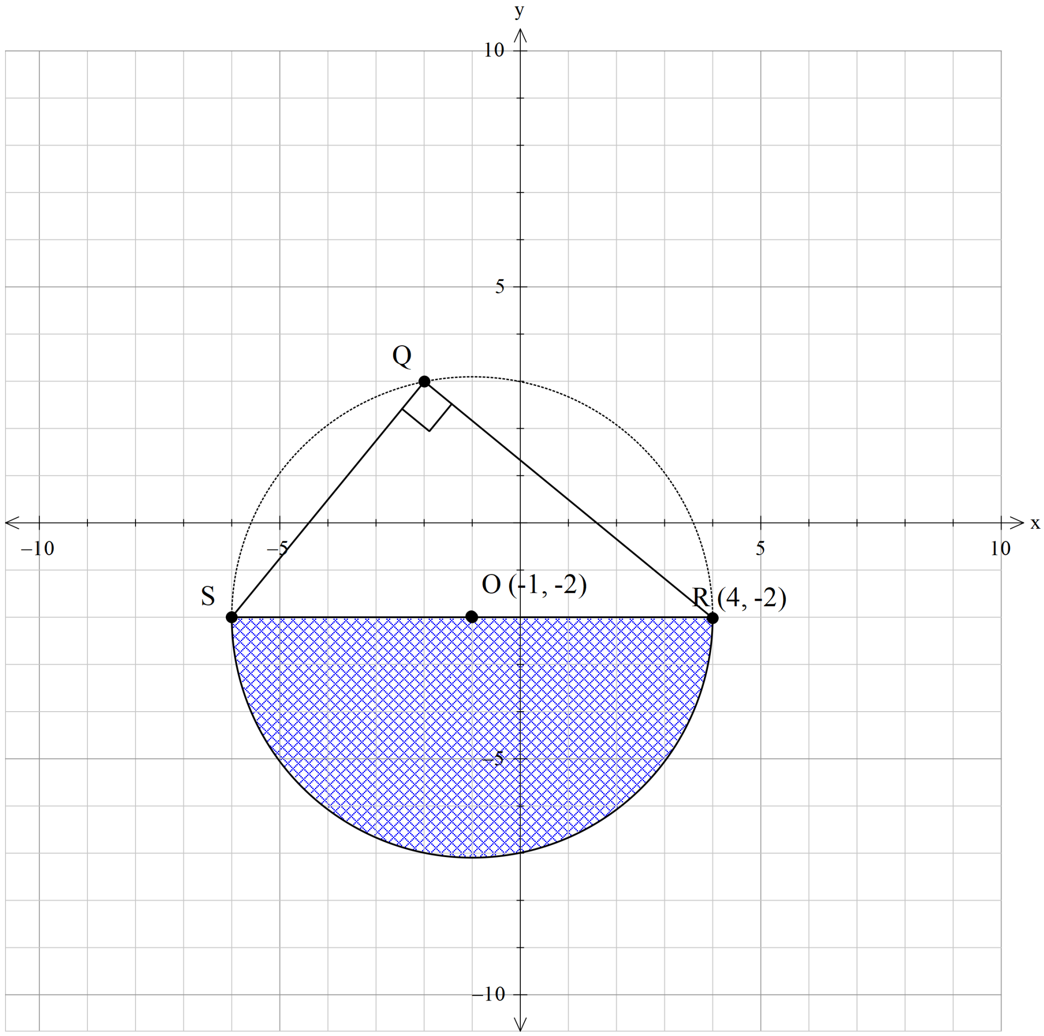
____

An isosceles triangle is drawn on each edge of the triangle. The height of each triangle is the same as the base length.

c) Draw these triangles and calculate the area of the triangles you have formed. What do you notice about the relationship between the areas? (5 marks)

3. The following diagram has been created by drawing a circle with a centre of (-1, -2) and a radius of 5 units. Thales' Theorem states that the triangle created by the diameter and any point on the circumference of the circle always has a right angle. **(15 marks)**

a) Marcela looks at the diagram and assumes the point Q must be (-2, 3). Use Pythagoras' Theorem to determine whether Marcela is correct. If she is incorrect, determine the correct ***y*-coordinate** of the point which intersects the circle at $x = -2$. (4 marks)



____

____

____

____

____

____

____

The blue shaded shape is a semicircle with a diameter the same as the hypotenuse.

b) Test to see whether the relationship between the areas of semicircles, formed using the diameter on all three edges follows Pythagoras' Theorem. Show all working, leaving your answers in exact value form where possible. Explain why this works. (11 marks)

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

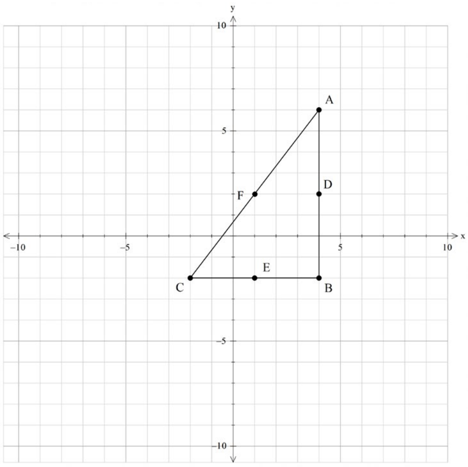
____

Formative assessment task – Checklist

Monitor each behaviour throughout the full scope of the assessment task. Rate each behaviour as follows: R – rarely, S – sometimes, C – consistently.

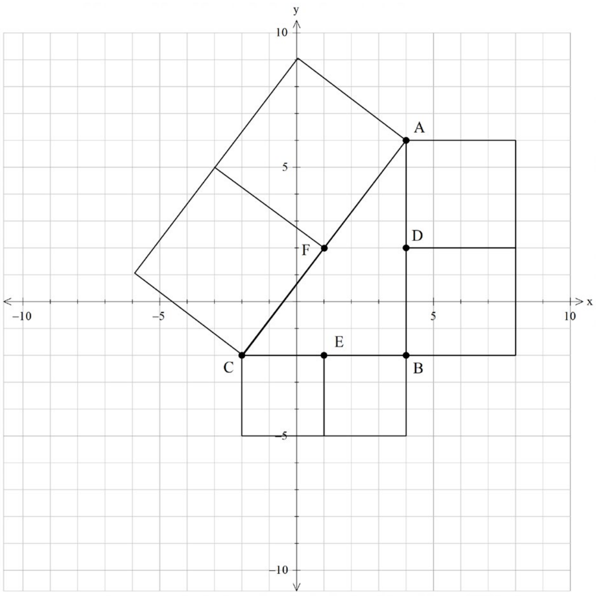
Name	Determines vertical and horizontal distances between given points	Determines midpoint of a line segment by inspection only	Calculates the distance between two given points	Calculates the midpoint of a line segment	Uses Pythagoras'Theorem to determine the length of the hypotenuse with diagram	Uses Pythagoras'Theorem to determine the length of the hypotenuse without	Solves problems involving distance on the Cartesian plane	Draws a labelled diagram to model familiar situations	Applies Pythagoras' Theorem to solve complex problems	Is the student above, at or below the expected Standard?

Sample Solution and Marking Key – Pythagoras and Beyond

Description	Marks
Question 1 a)	
$AB = 8$ units $BC = 6$ units $AC^2 = AB^2 + BC^2$ $= 8^2 + 6^2$ $AC = \sqrt{100} = 10$ units	
Determines the horizontal and vertical length AB and BC [1 each]	2
Applies Pythagoras' Theorem to determine the length of AC	1
Expresses the length AC using appropriate units	1
Question 1 b)	
 Midpoint AB, $D = \left(\frac{4+4}{2}, \frac{6+(-2)}{2}\right)$ $D = (4,2)$ Midpoint BC, $E = \left(\frac{4+(-2)}{2}, \frac{(-2)+(-2)}{2}\right)$ $E = (1,-2)$ Midpoint AC, $F = \left(\frac{4+(-2)}{2}, \frac{6+(-2)}{2}\right)$ $F = (1,2)$	
Labels the midpoint of AB, BC and AC on the diagram [1 each]	3
Uses the midpoint formula to calculate the midpoint	1

Sample Solution and Marking Key – Pythagoras and Beyond

Description	Marks
Question 1 c)	



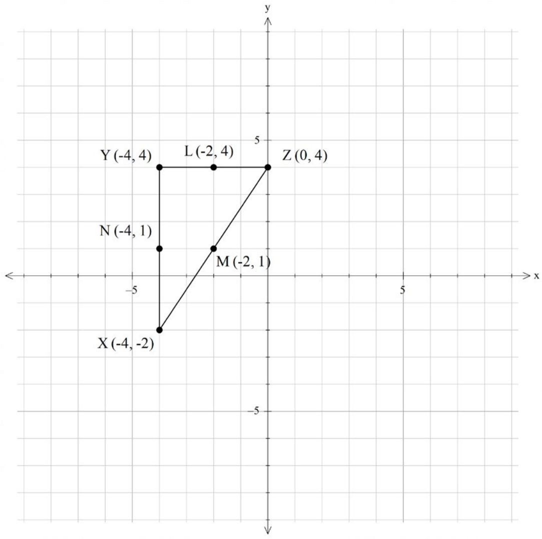
$$\text{Area (right)} = L \times W = 4 \times 8 = 32 \text{ units}^2$$

$$\text{Area (bottom)} = L \times W = 3 \times 6 = 18 \text{ units}^2$$

$$\text{Area (slant)} = L \times W = 5 \times 10 = 50 \text{ units}^2$$

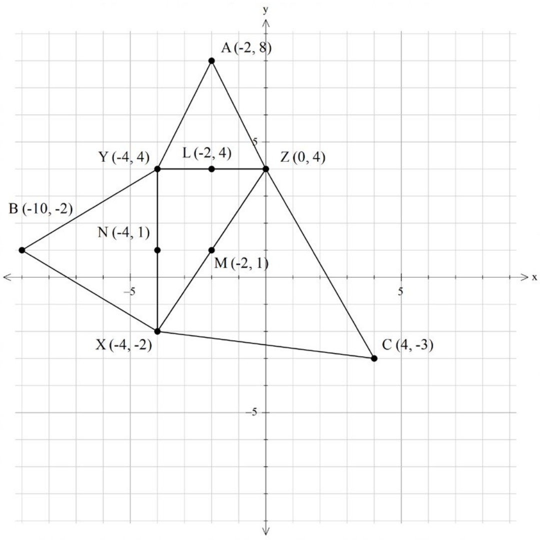
The sum of the area of two squares forming a rectangle on each of the short sides of a right-angled triangle is equal to the sum of the area of two squares forming a rectangle on the larger side. This follows the general pattern of Pythagoras' Theorem.

Description	Marks
Uses the midpoint to draw two squares on each side of the triangle [1 each side]	3
Calculates the area of each pair of squares [1 mark each]	3
Identifies that the area of the smaller shapes sums to the area of the larger shape	1
Compares this relationship to Pythagoras' Theorem	1
Subtotal	/16

Sample Solution and Marking Key – Pythagoras and Beyond	
Description	**Marks**
Question 2 a)	
	
Accurately plots the vertices of the triangle	1
Connects XY, YZ and ZY to plot triangle	1
Identifies the midpoint of each line segment using inspection	1
Question 2 b)	
$\text{XY} = 6 \text{ units}$ $\text{YZ} = 4 \text{ units}$ $\text{XZ}^2 = \text{XY}^2 + \text{YZ}^2$ $\text{XZ}^2 = 6^2 + 4^2$ $\text{XZ} = \sqrt{52} = 7.21 \text{ units}$	
Identifies the horizontal and vertical length XY and YZ [1 each]	2
Applies Pythagoras' Theorem to determine the length of XZ	1
Expresses the length XZ using appropriate units	1

Sample Solution and Marking Key – Pythagoras and Beyond

Description	Marks
Question 2 c)	

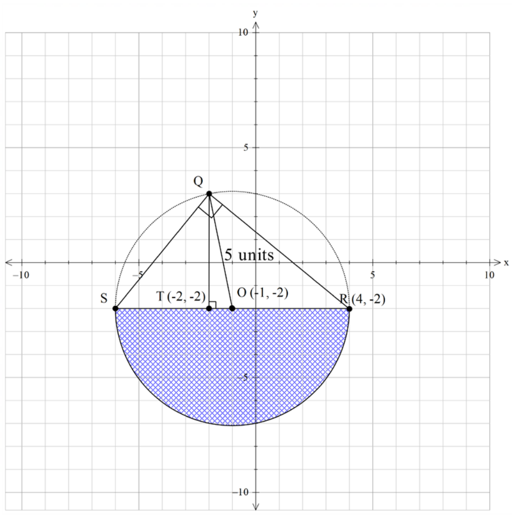


$$\text{Area } \Delta AYZ = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 4 \times 4$$
$$= 8 \text{ units}^2$$

$$\text{Area } \Delta BXY = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 6 \times 6$$
$$= 18 \text{ units}^2$$

$$\text{Area } \Delta CXZ = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 7.21 \times 7.21$$
$$= 26 \text{units}^2$$

Description	Marks
Draws apex of isosceles triangles perpendicular to midpoint	1
Calculates area of each triangle using area = $\frac{1}{2}$ × base × height [1 each]	3
Makes comparison between sum of area of smaller triangles and area of the large triangle	1
Subtotal	/12
Question 3 a)	



Marcela is incorrect because the hypotenuse of that triangle is 5 units, so the height cannot be 5 units.

$$OQ = 5 \text{ units}$$
$$OT = 1 \text{ unit}$$
$$QT^2 = OQ^2 - OT^2$$
$$QT^2 = 5^2 - 1^2$$
$$QT = \sqrt{24} = 4.90$$

Therefore, the coordinates of Q must be $(-2, -2 + 4.90)$ or $(-2, 2.9)$ and not $(-2, 3)$.

Description	Marks
Identifies that Marcela is not correct	1
Provides adequate reasoning why Marcela is not correct	1
Uses Pythagoras' Theorem to determine the vertical distance of line QT	1
Identifies the y-coordinate correctly	1

Sample Solution and Marking Key – Pythagoras and Beyond

Description	Marks
Question 3 b)	

Green circle

Diameter $= \text{QS}$

$\text{QS}^2 = 4^2 + 4.90^2$

$\text{QS}^2 = 16 + 24$

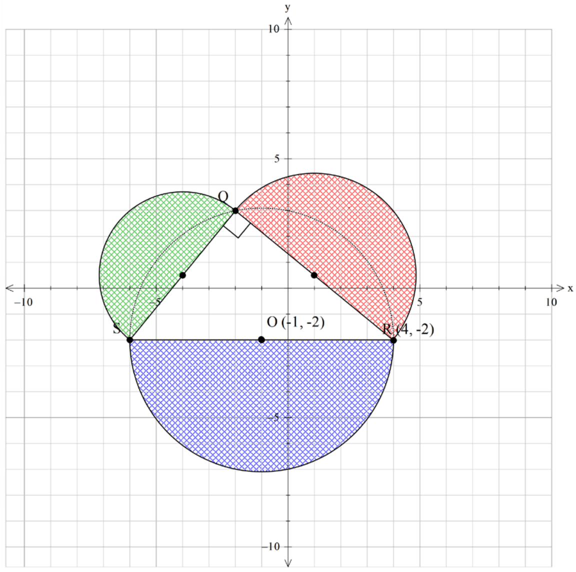
$\text{QS} = \sqrt{40} = 6.32 \text{ units}$

Radius $= 6.32 \div 2 = 3.16 \text{ units}$

Area $= \frac{1}{2} \times \text{r}^2 \times \pi$

$= \frac{1}{2} \times 3.16^2 \times \pi$

$= 5\pi = 15.71 \text{ units}^2$



Red circle

Diameter $= QR$

$\text{QR}^2 = 6^2 + 4.90^2$

$\text{QR}^2 = 36 + 24$

$\text{QR} = \sqrt{60} = 6.32 \text{ units}$

Radius $= 7.75 \div 2 = 3.87 \text{ units}$

Area $= \frac{1}{2} \times \text{r}^2 \times \pi$

$= \frac{1}{2} \times 3.16^2 \times \pi$

$= 7.5\pi = 23.56 \text{ units}^2$

Blue circle

Diameter = SR = 10 units

Radius = 5 units

Area $= \frac{1}{2} \times \text{r}^2 \times \pi$

$= \frac{1}{2} \times 5^2 \times \pi$

$= 12.5\pi = 39.27 \text{ units}^2$

$5\pi + 7.5\pi = 12.5\pi \text{ units}^2$

$15.71 + 23.56 = 39.27 \text{ units}^2$

Description	Marks
Uses Pythagoras' Theorem to calculate lengths QS and QR [1 each]	2
Uses QT value of 4.9 ($\sqrt{24}$) to calculate QS and QR	1
Uses QS and QR to determine radius of circles [1 each]	2
Calculates the area of semicircles using the appropriate radius [1 each]	3

Sample Solution and Marking Key – Pythagoras and Beyond	
Description	**Marks**
Expresses answers as exact values	1
Makes comparison between sum of area of smaller semicircles and area of the large semicircle	1
Relates this to Pythagoras' Theorem, discussing the $radius^2$ in each case	1
Subtotal	/15
Total	**43**

Appendix C:
Summative Assessment Task

Pythagoras' TV-rem

Appendix C | Summative assessment task

Title of task

Pythagoras' TV-rem

Task details

Description of task	Students investigate and apply coordinate geometry and Pythagoras' Theorem in relation to TV related questions.
Type of assessment	Summative assessment
Purpose of assessment	To give the students an opportunity to demonstrate their knowledge of Pythagoras' Theorem and coordinate geometry To give students an opportunity to demonstrate their ability to apply these skills in familiar and unfamiliar real-life application
Assessment strategy	Individual summative assessment in test conditions. Students can receive support or guidance from teacher, but note this on their work.
Evidence to be collected	Individual student task sheet
Suggested time	One lesson in class

Content description

Content from the Western Australian curriculum

- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles

Task preparation

Prior learning

Students are in the last week of their unit on Number and Algebra, and Measurement and Geometry. Students have developed their skills to determine the midpoint of two points located on the Cartesian plane, the gradient of two points on the Cartesian plane and the distance between two points, using the distance formula and Pythagoras' Theorem.

Assessment differentiation

Teachers should differentiate their teaching and assessment to meet the specific learning needs of their students, based on their level of readiness to learn and their need to be challenged.
Where appropriate, teachers may either scaffold or extend the scope of the assessment tasks.

Assessment task

Assessment conditions

Test conditions, no collaboration between students

Resources

- calculators
- ruler

Instructions to teacher

This summative assessment is in the form of a response task. Students work individually under test conditions to respond to the prompts to demonstrate their level of understanding of Pythagoras' Theorem and coordinate geometry. The questions allow students to explore the applications of Pythagoras' Theorem in relation to the manufacture and sale of televisions.

Provide appropriate support to students where required, and indicate this on their work. For students who have low literacy skills, scaffold questions 3 and 4 to allow them to access the Mathematics appropriately.

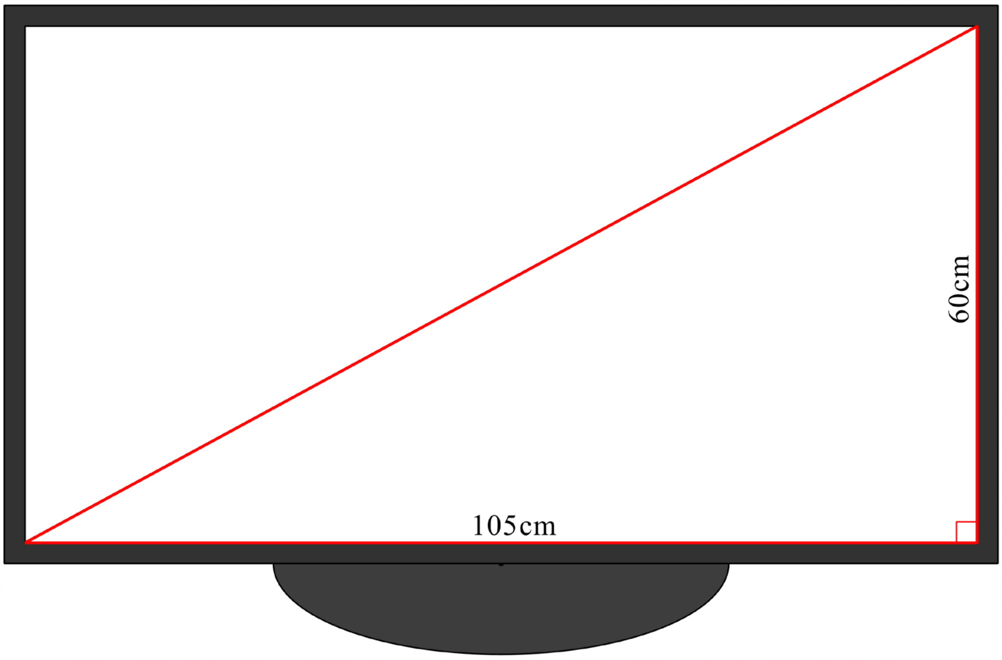
This assessment will provide opportunities for students to demonstrate a range of specific skills. The way in which students apply these skills will help to determine the achievement of students in this unit. An example of this can be observed in Question 2d. A student who is working above the expected Standard will be able to identify that the lines DE and EF are the same length, as E is the midpoint of line DF. Similarly, they will know that CE is half of AD, as they are part of two sides on a parallelogram. Observations of how the students have achieved their solution, rather than just what the solution is, allow for a fine-grained examination of the behaviours and achievement of students in this unit.

Pythagoras' TV-rem – Task sheet

1. Televisions have an advertised size, which is determined by the length from one corner of the screen to the opposite corner (rounded to the nearest cm). This measurement is of the screen only and does not include any part of the plastic frame which holds the screen in place and houses the electronic components.

   Determine the advertised size, the width or the height as required of the television screens pictured. **(5 marks)**

   a) (1 mark)



___

___

___

___

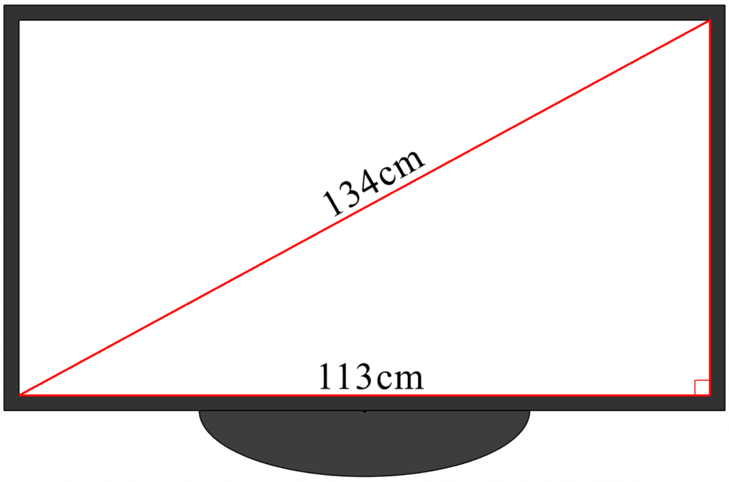
___

___

___

___

b) (2 marks)



____

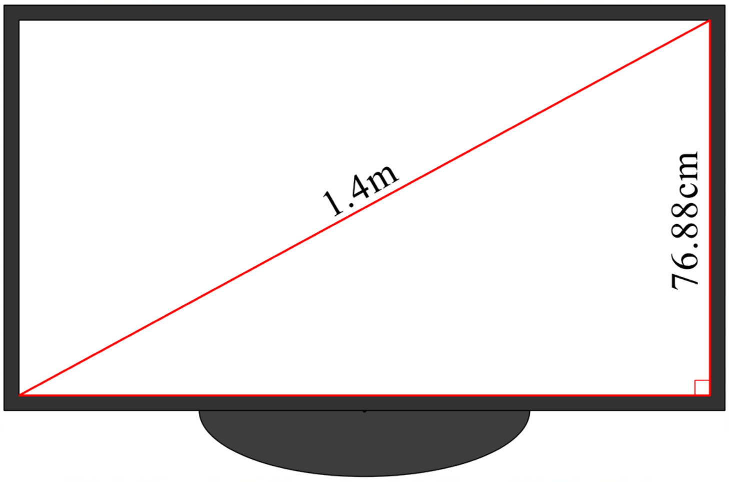
____

____

____

____

c) (2 marks)



____

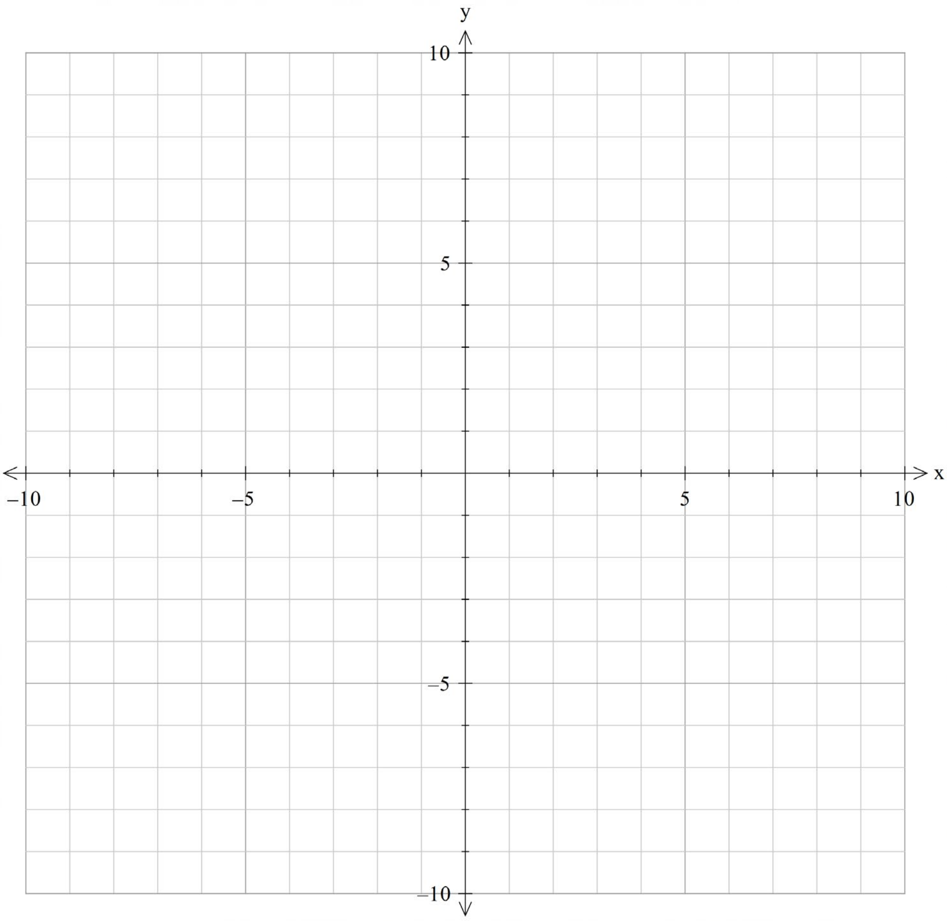
____

____

____

____

2. Follow the instructions below. **(17 marks)**



a) Plot A $(-4, 6)$, B $(6, 6)$, C $(2, -2)$ and D $(-8, -2)$ and draw the resulting parallelogram.

(2 marks)

b) Plot and label E, the midpoint of $\overline{BC}$, then determine the length and gradient of $\overline{DE}$.

(4 marks)

____

____

____

____

____

____

____

c) Point E is also the midpoint of $\overline{DF}$. Determine the coordinates of F. Explain how you got this coordinate. (3 marks)

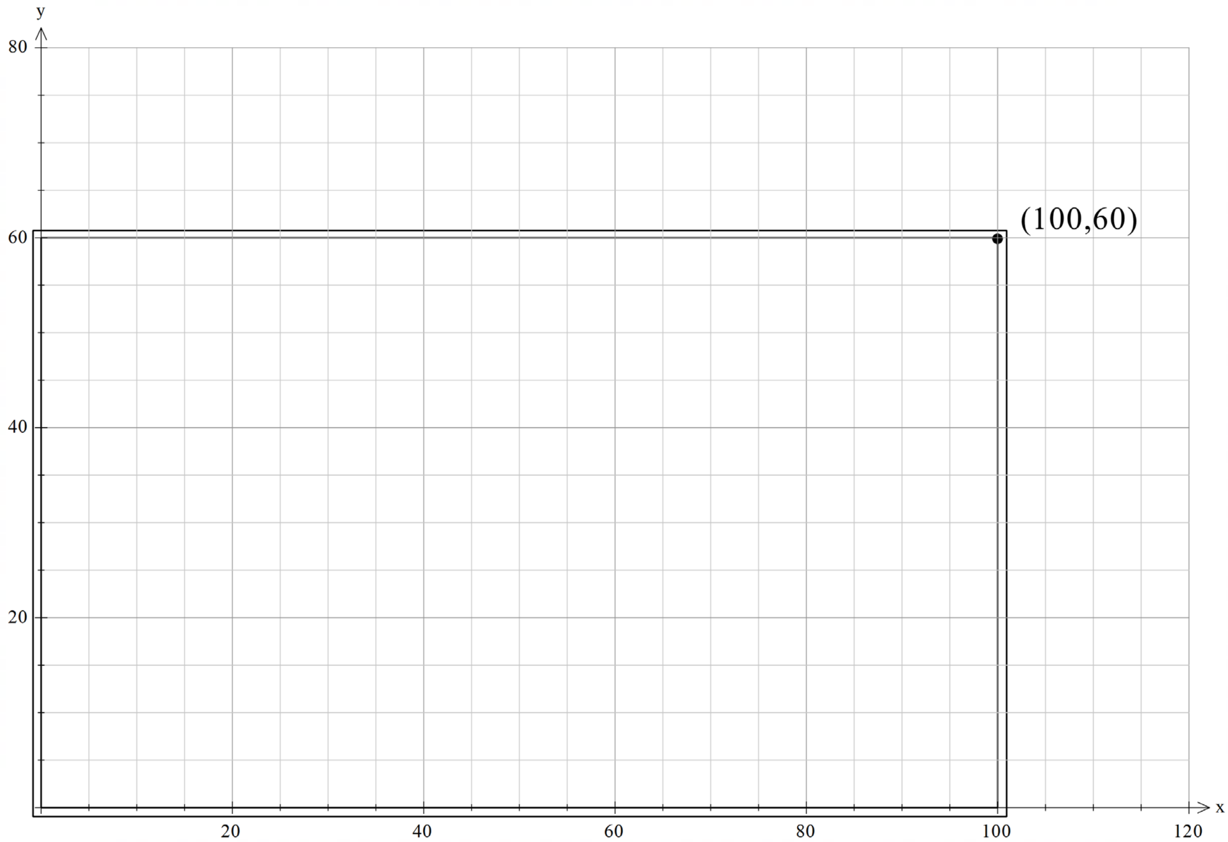
d) When F is connected to B, it forms an irregular pentagon, AFECD. What is the perimeter of this pentagon? (8 marks)

3. The blueprint of a television has been placed on a Cartesian plane, each unit representing 1 cm. The power cord and the TV aerial at the back of the television need to be placed appropriately.

(16 marks)

- To place the aerial, the midpoint of the top edge of the television screen is marked, A. The aerial is connected at the midpoint of the line segment which spans from A to the bottom right corner of the television.
- To place the power cord, the midpoint of A and the top right corner is marked, B. A line is then drawn from B to the bottom right corner of the television. The midpoint of this line is where the power cord is placed.

a) Represent the location of these two cables on the diagram below, clearly labelling all relevant points. Show your calculations in the space provided below. (6 marks)



____

____

____

____

____

____

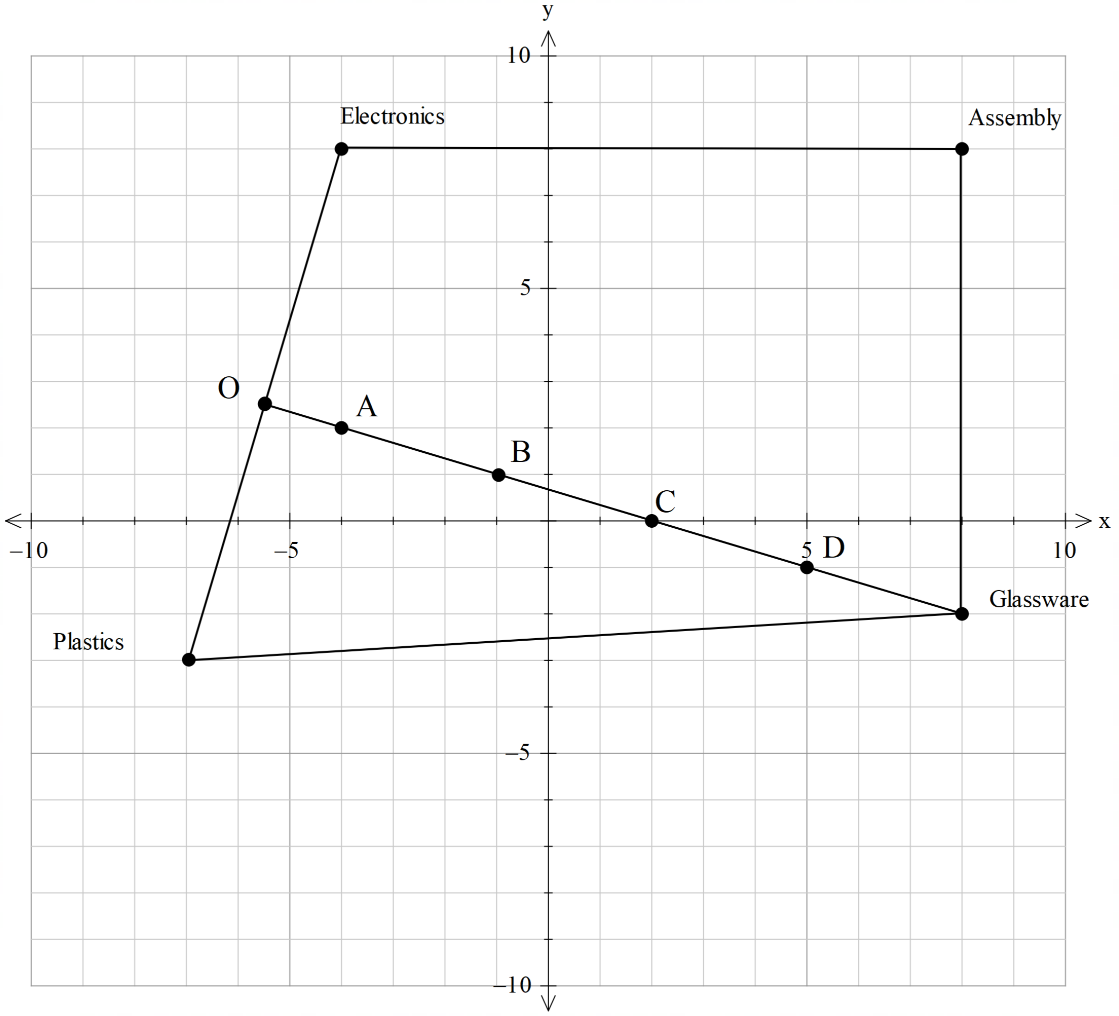
b) The power cable connects inside the television to a light at $(0,0)$, which indicates whether the power is on or off. Sam wants to use Pythagoras' Theorem to determine this length, while Leith wants to use coordinate geometry to determine the length. Show how they will get the same solution using either approach. (4 marks)

c) Is the triangle formed by $(0,0)$, $(100,0)$ and the location of your power cable a right-angled triangle? Support your answer with appropriate evidence. (6 marks)

4. Televisions are comprised of 3 main parts: **(18 marks)**

- the glass screen
- the plastic casing
- the electronics.

These parts are all made in separate facilities and then transported to one warehouse where they are assembled and then sent for delivery to retail outlets. The manufacturers want to move the existing Assembly point so it reduces the total distance travelled for all of their component parts to reduce their environmental footprint. To do this, they will relocate the Assembly point to one of the line coordinates A, B, C or D.



a) Determine the location of point O, the midpoint of the electronics and plastics factories. (3 marks)

____

____

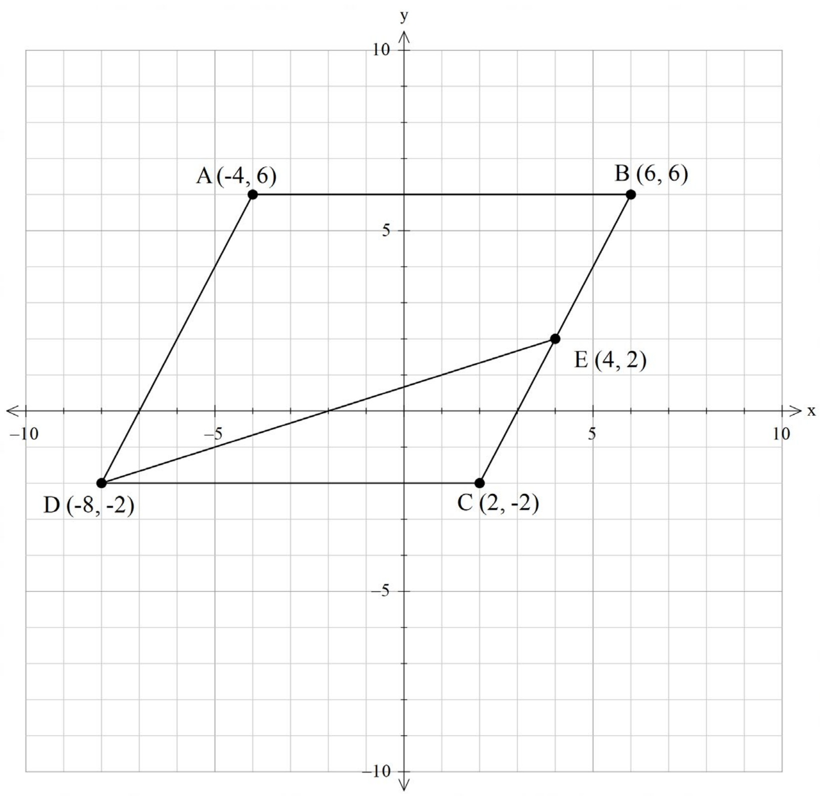
____

____

b) Calculate the gradient and length of the line from point O to the glassware factory.

(3 marks)

c) Determine the new location of the Assembly point which will reduce the travel time the most. How much shorter is this point? Justify your choice. (12 marks)

Sample Solution and Marking Key – Pythagoras' TV-rem	
Description	**Marks**
Question 1	
a) Advertised $\text{size}^2 = 60^2 + 105^2 = 3600 + 11025$ Advertised size $= 120.9\text{cm} \approx 121$ cm b) $\text{Height}^2 = 134^2 - 113^2 = 17956 - 12769$ Height $= 72.02$ cm ≈ 72 cm c) $\text{Width}^2 = 140^2 - 76.88^2 = 19600 - 5910.53$ Width $= 117.00 \approx 117$ cm	
Calculates the length of the hypotenuse from a diagram using Pythagoras' Theorem	1
Identifies shorter length is found through subtraction	1
Calculates the shorter lengths using Pythagoras' Theorem (1 each)	2
Rounds answers appropriately	1
Subtotal	/5
Question 2	
a) 	
Plots points accurately	1
Draws parallelogram	1

Sample Solution and Marking Key – Pythagoras' TV-rem

Description	Marks

b)

Midpoint of BC, E $= \left(\frac{6+2}{2}, \frac{6+(-2)}{2}\right)$

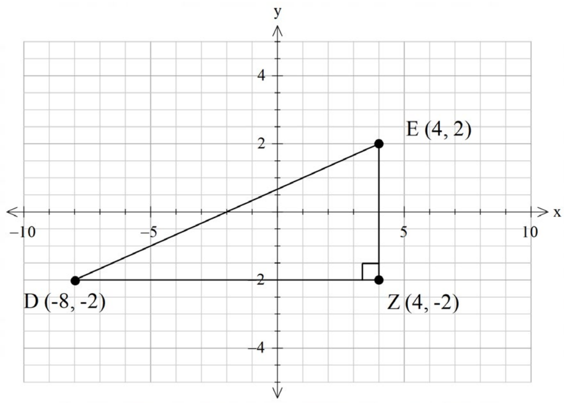
$E = (4,2)$ (shown above)

$DE^2 = DZ^2 + EZ^2$

$DE^2 = \sqrt{\left(4-(-8)\right)^2 + \left(2-(-2)\right)^2}$

$DE = \sqrt{160} = 12.65$ units

gradient $\overline{DE} = \frac{(2-(-2))}{4-(-8)} = \frac{4}{12} = \frac{1}{3}$



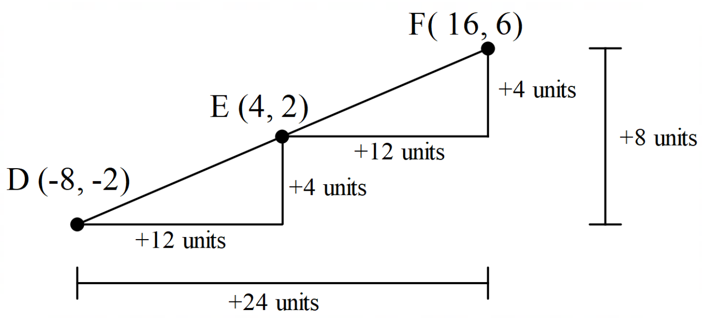
Clearly plots and labels Point E	1
Identifies vertical and horizontal change from D to E	1
Calculates length DE using distance formula or Pythagoras' Theorem	1
Calculates gradient of DE using vertical and horizontal change	1

c)

$(4,2) = \left(\frac{-8+x}{2}, \frac{-2+y}{2}\right)$

$8 = -8 + x$
$x = 16$

$4 = -2 + y$
$y = 6$



Therefore, F (16, 6)

Uses an appropriate approach to determine the *x*-coordinate of F	1
Uses an appropriate approach to determine the *y*-coordinate of F	1
Explains their answer, justifying the mathematics used (any appropriate approach)	1

d)

Length AF = 20 units

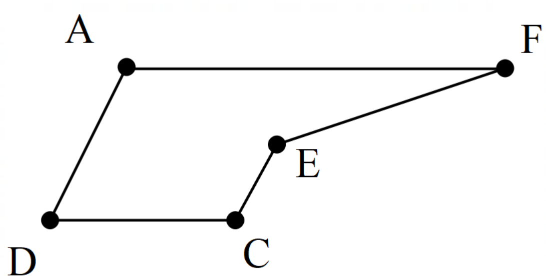
Length DC = 10 units

Length EF = Length DE = 12.65 units

Length $CE^2 = 2^2 + 4^2 = 20$

Length CE = $\sqrt{20} = 4.472$ units

Length AD = 2 x CE = 2 x 4.472 = 8.944



Total perimeter = 20 + 10 + 12.65 + 4.472 + 8.944 = 56.06 units

Determines the length of horizontal lines, AF and DC [1 each]	2

Sample Solution and Marking Key – Pythagoras' TV-rem

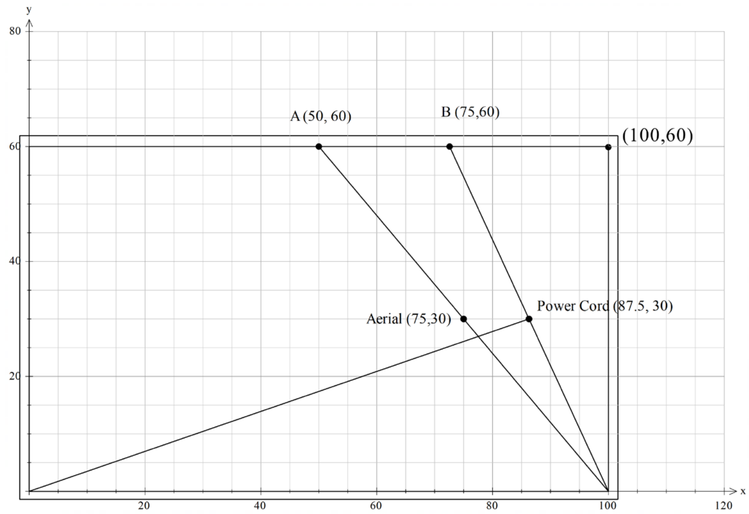
Description	Marks
Determines the length of EF using an appropriate method	1
Determines the length of CE using Pythagoras' Theorem or distance formula	1
Determines the length of AD using appropriate method	1
Recognises that DE = EF, or that CE is $\frac{1}{2}$ of AD, using this to calculate the lengths	1
Calculates the total perimeter using all previous measurements	1
Includes appropriate units and rounding	1
Subtotal	/17

Question 3

a)

$$\text{Aerial} = \left(\frac{50+100}{2}, \frac{(60+0)}{2}\right)$$
$$= (75, 30)$$

$$\text{Power Cord} = \left(\frac{75+100}{2}, \frac{60+0}{2}\right)$$
$$= (87.5, 30)$$



Description	Marks
Determines the location of A and B from the statement [1 mark] with no support [1 mark]	2
Locates the midpoint of both line segments (graphically or algebraically) [1 each]	2
States the coordinates of the aerial [1 mark] and the power cord [1 mark]	2

b)

Coordinates of the light, (0, 0).

Coordinates of the power cable (87.5, 30)

Sam's approach

$$\text{length}^2 = 87.5^2 + 30^2$$
$$\text{length}^2 = 7656.25 + 900$$
$$\text{length} = \sqrt{8556.25} = 92.5\text{cm}$$

Leith's approach

$$\text{length} = \sqrt{(87.5-0)^2 + (30-0)^2}$$
$$\text{length} = \sqrt{7656.25 + 900}$$
$$\text{length} = \sqrt{8656.25} = 92.5\text{cm}$$

Both methods yield the same result.

Sample Solution and Marking Key – Pythagoras' TV-rem

Description	Marks
Identifies the vertical and horizontal components of the right-angled triangle formed	1
Uses Pythagoras' Theorem and distance formula to calculate the length [1 each]	2
Compares these two values, indicating they are the same	1

c)

If right angled, then

$BC^2 = AC^2 + AB^2$

$AC^2 = 87.5^2 + 30^2$

$AC = 92.5 \text{ cm(from b)}$

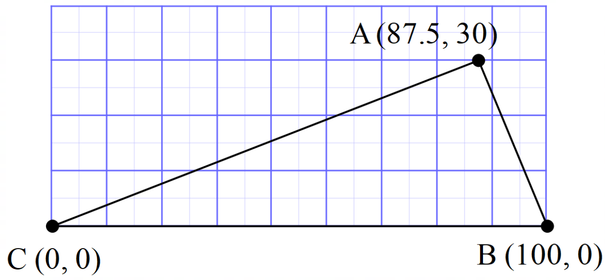
$AB^2 = 12.5^2 + 30^2$

$AB = \sqrt{1056.25} = 32.5 \text{ cm}$

$BC^2 = 92.5^2 + 32.5^2$

$BC^2 = 9612$

$BC = \sqrt{9612} = 98.04 \text{ cm} \neq 100 \text{ cm}$



Therefore, not a right-angled triangle

Description	Marks
Identifies Pythagoras' Theorem required to check for a right angle	1
Identifies BC as longest side	1
Calculates the length of AB using an appropriate method	1
Calculates the length of AC using an appropriate method	1
Compares calculated length of BC using Pythagoras to actual length	1
Identifies the triangle is not right-angled	1
Subtotal	/16

Question 4

a)

Coordinates of $O = \left(\frac{-4+(-7)}{2}, \frac{8+(-3)}{2}\right)$

$O = (-5.5, 2.5)$

Description	Marks
Identifies the coordinates of electronics and plastics factories	1
Uses coordinates to determine the midpoint, O [1 mark each coordinate]	2

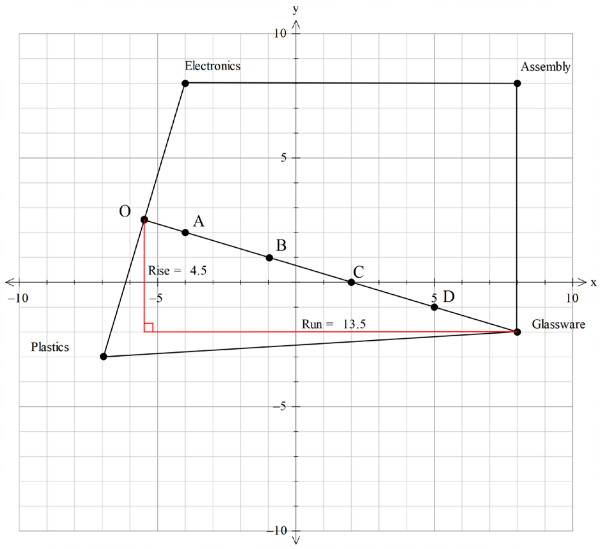
Sample Solution and Marking Key – Pythagoras' TV-rem

Description	Marks

b)

$$\text{Gradient } \overline{OG} = \frac{2.5-(-2)}{-5.5-8}$$
$$= \frac{4.5}{-13.5}$$
$$= -\frac{1}{3}$$

$$\text{Distance } \overline{OG}^2 = 4.5^2 + 13.5^2$$
$$= 20.25 + 182.25$$
$$= 202.75$$
$$OG = \sqrt{202.75}$$
$$= 14.24 \text{ units}$$



Description	Marks
Determines magnitude of gradient	1
Determines direction of gradient	1
Calculates the distance, $\overline{OG}$, using an appropriate method	1

c)

Current distance = EA +GA + PA

EA = 12 units

GA = 10 units

PA = PE + EA or PG + GA

Option 1: PE + EA

$PE = \sqrt{11^2 + 3^2}$

$PE = \sqrt{130} = 11.40$ units

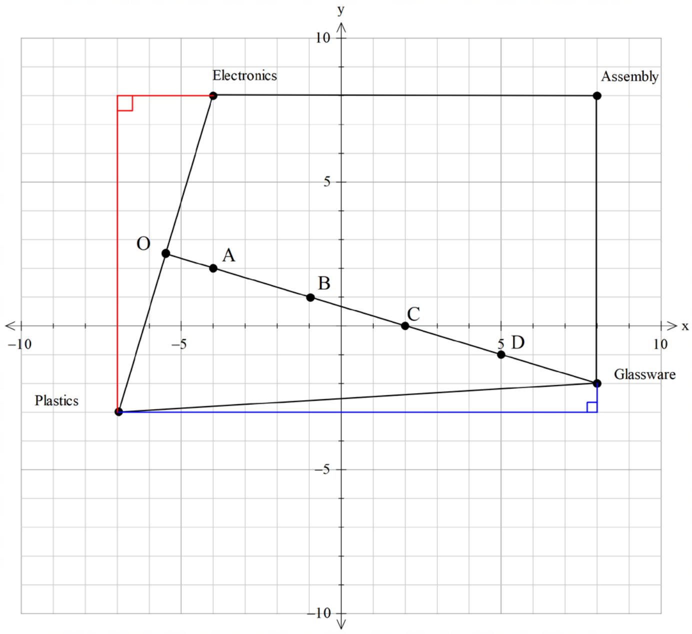
Total $= 11.40 + 12 = 23.40$ units

Option 2: PG + GA

$PG = \sqrt{15^2 + 1^2}$

$PG = \sqrt{226} = 15.03$ units

Total $= 15.03 + 10 = 25.03$ units



Therefore, currently, the total distance is 10 + 12 + 23.40 = 45.40 units.

Check end points, A and D

Point A: Total distance = OE + OP + 2OA + AG

Point D: Total distance = OE + OP + 2OD + DG

Only need to compare last 2 terms as first ones will be the same.

Sample Solution and Marking Key – Pythagoras' TV-rem

Description

Point A: 2OA + AG

Point D: 2OD + DG

$$\mathrm{OA} = \sqrt{((-4) - (-5.5))^2 + (2 - 2.5)^2}$$
$$\mathrm{OA} = \sqrt{1.5^2 + 0.5^2} = \sqrt{2.5} = 1.58$$
$$\mathrm{AG} = \mathrm{OG} - \mathrm{OA} = 14.24 - 1.58 = 13.64$$
$$2\mathrm{OA} + \mathrm{AG} = 2 \times 1.58 + 13.64 = 16.8$$

$$\mathrm{OD} = \sqrt{((-4) - (5))^2 + (2 - (-1))^2}$$
$$\mathrm{OD} = \sqrt{9^2 + 3^2} = \sqrt{90} = 9.49$$
$$\mathrm{DG} = \mathrm{OD} - \mathrm{OD} = 14.24 - 9.49 = 4.75$$
$$2\mathrm{OD} + \mathrm{AD} = 2 \times 9.49 + 4.75 = 23.73$$

Total distance to Point A is $\mathrm{PE}\ (\mathrm{PO} + \mathrm{EO}) + 2\mathrm{OA} + \mathrm{AG} = 11.4 + 16.8 = 28.2$ units

This is shorter than the existing distance by 27.2 units.

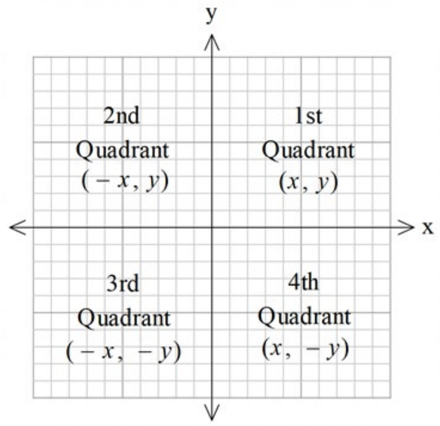
Description	Marks
Determines the horizontal and vertical component of the current location [1 each]	2
Identifies two possible routes from plastics to assembly	1
Determines best route from plastics to assembly	1
Uses previous distances to determine the total distance to the current location	1
Identifies the best location	1
Calculates the distance to this location	1
Compares this to the distance to the current location	1
Provides justification of answer Possible considerations (choose up to four where relevant) • Each option contained distance PO and OE, which is the same as PE • Knowing the distance $\overline{\mathrm{DG}}$ can be used to determine the distance $\overline{\mathrm{CG}}, \overline{\mathrm{BG}}, \overline{\mathrm{AG}}$ $\overline{\mathrm{OG}} = 14.24$ units, from b) $\overline{\mathrm{OG}} = \overline{\mathrm{OA}} + \overline{\mathrm{AG}}$, i.e. $\overline{\mathrm{OG}} - \overline{\mathrm{AG}} = \overline{\mathrm{OA}}$ • The distance from O is travelled twice, by plastics and electronics. This only needs to be calculated once • Due to the distance above being calculated twice, it should be minimised. • OA:OB = 1:3, OA:OC = 1:5, OA:OD = 1:7 • Any other reasoning or justification as appropriate	4
Subtotal	/18
Total	**56**

Glossary

Term	Meaning
Cartesian coordinate system	Two intersecting number lines are taken intersecting at right angles at their origins to form the axes of the coordinate system. The plane is divided into four quadrants by these perpendicular axes called the x-axis (horizontal line) and the y-axis (vertical line). The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These ordered are called the coordinates of the point. This is called the Cartesian coordinate system. The plane is called the Cartesian plane.
gradient	If A(x_1, y_1) and B(x_2, y_2) are points in the plane, $x_2 - x_1 \neq 0$, the gradient of the line segment (interval) AB $= \frac{\text{rise}}{\text{run}} = \frac{x_2 - x_1}{y_2 - y_1}$. The gradient of a line is the gradient of any line segment (interval) within the line.
horizontal distance/run	The horizontal distance is the change in the x-coordinates between the points A(x_1, y_1) and B(x_2, y_2). The horizontal distance is calculated as $x_2 - x_1$. It is commonly used to determine the distance between two points and the gradient of the line segment.
hypotenuse	The hypotenuse is the longest side in a right-angled triangle. The hypotenuse always occurs opposite the right angle.
line segment/interval	If A and B are two points on a line, the part of the line between and including A and B is called a line segment or interval. The distance AB is a measure of the size or length of AB.
midpoint	The midpoint M of a line segment (interval) AB is the point that divides the segment into two equal parts. Let A(x_1, y_1) and B(x_2, y_2) be points in the Cartesian plane. Then the midpoint M of line segment AB has coordinates $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.
negative gradient	A negative gradient means that a line segment decreases as the graph moves from left to right, as x increases, y decreases. The value of the gradient is less than zero. It occurs when the rise and the run are opposite signs.
number line	A number line gives a pictorial representation of real numbers.

Term	Meaning
ordered pair	A ordered pair refers to the two numbers which make up a Cartesian coordinate. The order of these numbers is important. For example, $(2, 4)$ is explicitly different from $(4, 2)$.
origin	The origin is the point on the Cartesian plane where the two axes intersect. It has the coordinates $(0, 0)$ and every coordinate (x, y) is considered x units right and y units up from the origin.
plot	To plot is to accurately place the location specific coordinates.
point	A point marks a position, but has no size.
positive gradient	A positive gradient means that a line segment rises as the graph moves from left to right, as x increases, y also increases. The value of the gradient is greater than zero. It occurs when the rise and the run are both the same sign.
Pythagoras' Theorem	For a right-angled triangle The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides. In symbols, $c^2 = a^2 + b^2$. If $c^2 = a^2 + b^2$ in a triangle ABC, then C is a right angle.

<table>
<tr><th>Term</th><th>Meaning</th></tr>
<tr><td>quadrant</td><td>The quadrants represent the four planes that a Cartesian plane is split into by the axes.

Where x and y are positive real numbers:
• the first quadrant contains all coordinates in the form (x, y)
• the second quadrant contains all coordinates in the form $(-x, y)$
• the third quadrant contains all coordinates in the form $(-x, -y)$
• the fourth quadrant contains all the coordinates in the form $(x, -y)$.


Any points that lie on either axis are not considered to be in any quadrant.</td></tr>
<tr><td>undefined gradient</td><td>A line with an undefined gradient represents a vertical line. This occurs when the horizontal distance between two points on a line segment is zero. Lines with an undefined gradient are parallel to the y-axis.</td></tr>
<tr><td>vertical distance/rise</td><td>The vertical distance is the change in the y-coordinates between the points A(x_1, y_1) and B(x_2, y_2) The vertical distance is calculated as $y_2 - y_1$. It is commonly used to determine the distance between two points and the gradient of the line segment.</td></tr>
<tr><td>x-axis</td><td>On the Cartesian plane, the number line making up the horizontal axis is the x-axis. The x-coordinate is the first coordinate listed in the ordered pair (x, y).</td></tr>
<tr><td>y-axis</td><td>On the Cartesian plane, the number line making up the vertical axis is the y-axis. The y-coordinate is the second coordinate listed in the ordered pair (x, y)</td></tr>
<tr><td>zero gradient</td><td>A line with a gradient of zero represents a horizontal line. This occurs when the vertical distance between two points on a line segment is zero. Lines with a gradient of zero are parallel to the x-axis.</td></tr>
</table>

Acknowledgements

Lesson sequence

Lesson 4	Map adapted from: © OpenStreetMap contributors. (n.d.). [Map of street grid in Wembley, Perth]. Retrieved June, 2021, from https://www.openstreetmap.org/#map=17/-31.93738/115.81433 Used under an Open Data Commons Open Database licence.

Appendix A

Appendix A.9	Exit ticket template adapted from: Clker-Free-Vector-Images. (2012). [Graphic of green exit symbol]. Retrieved July, 2021, from https://pixabay.com/vectors/fire-safety-signs-symbols-exit-40631/
Appendix A.11	Slope 1 adapted from: Vieli, J. (2017). [Photograph of a snowy mountainscape with a blue sky]. Retrieved June, 2021, from https://pixabay.com/photos/winter-mountains-snow-landscape-4680713/ Slope 2 adapted from: Gaida, M. (2021). [Photograph of two skiers in the snow]. Retrieved June, 2021, from https://pixabay.com/photos/cross-country-skiing-skiers-ski-5908416/ Slope 3 adapted from: van de Wal, R. (2015). [Photograph of a person wearing white and red goggles skiing downhill]. Retrieved June, 2021, from https://pixabay.com/photos/skiing-girl-sun-snow-winter-ski-1723857/ Slope 4 adapted from: miaalthoff. (2017). [Photograph of a person wearing with black and white goggle skiing downhill]. Retrieved June, 2021, from https://pixabay.com/photos/snow-winter-sport-skier-mountain-3090067/ Slope 5 adapted from: Carli, M. (2017). [Photograph of tall trees on a snowy slope]. Retrieved June, 2021, from https://pixabay.com/photos/winter-snow-snow-covered-wintry-2949606/ Slope 6 adapted from: Simon. (2017). [Photograph of a chairlift on a steep snowy slope]. Retrieved June, 2021, from https://pixabay.com/photos/chairlift-alpine-skiing-skiing-ski-2080001/ Slope 7 adapted from: Braxmeier, H. (2012). [Photograph of a group of people hiking in the snow]. Retrieved June, 2021, from https://pixabay.com/photos/backcountry-skiiing-winter-hike-hike-16154/ Slope 8 adapted from: Westendarp, E. (2017). [Photograph of a chairlift and people skiing]. Retrieved June, 2021, from https://pixabay.com/photos/winterberg-north-slope-hochsauerland-1961027/ Slope 9 adapted from: Kofler, P. (2016). [Photograph of a person wearing a green helmet skiing]. Retrieved June, 2021, from https://pixabay.com/photos/skiers-ski-runway-skiing-winter-1274666/ Slope 10 adapted from: moritz320. (2009). [Photograph of snowy mountains behind a small group of buildings]. Retrieved June, 2021, from https://pixabay.com/photos/winter-mountains-snow-wintry-1159196/

Features of a slope image adapted from: Walkerssk. (2015). [Photograph of a snowy mountainscape with yellow clouds in the sky]. Retrieved June, 2021, from https://pixabay.com/photos/alps-mountain-mountains-snow-1368034/

Model of a ski slope image adapted from: OpenClipart-Vectors. (2014). [Graphic of snowy mountain]. Retrieved June, 2021, from https://pixabay.com/vectors/hill-mountain-snow-snowclad-rock-575621/

The gradient image adapted from: OpenClipart-Vectors. (2014). [Graphic of steep snowy mountain]. Retrieved June, 2021, from https://pixabay.com/vectors/hill-mountain-snow-snowclad-nature-575620/

Appendix A.12 Map of Australia adapted from: Clker-Free-Vector-Images. (2012). [Outline of Australia with state/territory borders]. Retrieved June, 2021, from https://pixabay.com/vectors/australia-continent-geography-map-23497/

Appendix A.13 Maps of Tasmania adapted from: Clker-Free-Vector-Images. (2012). [Outline of Tasmania]. Retrieved June, 2021, from https://pixabay.com/vectors/tasmania-map-australia-island-23533/